Markscheme

May 2016

Mathematics

Standard level

Paper 2
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Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid Method; working must be seen.

(M) Marks awarded for a valid Method; may be implied by correct subsequent working.

A Marks awarded for an Answer or for Accuracy; often dependent on preceding M marks.

(A) Marks awarded for an Answer or for Accuracy; may be implied by correct subsequent working.

R Marks awarded for clear Reasoning.

N Marks awarded for correct answers if no working shown.

AG Answer given in the question and so no marks are awarded.

Using the mark scheme

1 General

Mark according to RM assessor instructions

2 Method and Answer/Accuracy marks

- Do not automatically award full marks for a correct answer; all working must be checked, and marks awarded according to the mark scheme.
- It is generally not possible to award M0 followed by A1, as A mark(s) depend on the preceding M mark(s), if any. An exception to this rule is when work for M1 is missing, as opposed to incorrect (see point 4).
- Where M and A marks are noted on the same line, eg M1A1, this usually means M1 for an attempt to use an appropriate method (eg substitution into a formula) and A1 for using the correct values.
- Where there are two or more A marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award A0A1A1.
- Where the markscheme specifies (M2), N3, etc., do not split the marks, unless there is a note.
- Most M marks are for a valid method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.

Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final A1.

3 N marks

If no working shown, award N marks for correct answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (M, A, R). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award N0.

- Do not award a mixture of N and other marks.
- There may be fewer N marks available than the total of M, A and R marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the N marks and the implied marks. There are times when all the marks are implied, but the N marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
4 Implied and must be seen marks

*Implied marks appear in brackets eg (M1).*
- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the N marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (M1) followed by A1 for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (M1).

*Must be seen marks appear without brackets eg M1.*
- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to M0 or A0 for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (FT) marks are awarded where an incorrect answer (final or intermediate) from one part of a question is used correctly in subsequent part(s) or subpart(s). Usually, to award FT marks, there must be working present and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the final answer, then FT marks should be awarded if appropriate. Examiners are expected to check student work in order to award FT marks where appropriate.*

- Within a question part, once an error is made, no further A marks can be awarded for work which uses the error, but M and R marks may be awarded if appropriate. (However, as noted above, if an A mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the mark scheme.
- If the question becomes much simpler because of an error then use discretion to award fewer FT marks.
- If the error leads to an inappropriate value (eg probability greater than 1, use of \( r > 1 \) for the sum of an infinite GP, \( \sin \theta = 1.5 \), non integer value where integer required), do not award the mark(s) for the final answer(s).
- The mark scheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a “show that” question, if an error in a previous subpart leads to not showing the required answer, do not award the final A1. Note that if the error occurs within the same subpart, the FT rules may result in further loss of marks.
6 Mis-read

If a candidate incorrectly copies information from the question, this is a mis-read (MR). A candidate should be penalized only once for a particular mis-read. Use the MR stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an M mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.

- If the question becomes much simpler because of the MR, then use discretion to award fewer marks.
- If the MR leads to an inappropriate value (eg probability greater than 1, use of \( r > 1 \) for the sum of an infinite GP, \( \sin \theta = 1.5 \), non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates’ own work does not constitute a misread, it is an error.

7 Discretionary marks (d)

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief note written next to the mark explaining this decision.

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by METHOD 1, METHOD 2, etc.
- Alternative solutions for parts of questions are indicated by EITHER . . . OR. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

Unless the question specifies otherwise, accept equivalent forms.

- As this is an international examination, accept all alternative forms of notation.
- In the markscheme, equivalent numerical and algebraic forms will generally be written in brackets immediately following the answer.
- In the markscheme, simplified answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics SL guide says: Students must always use correct mathematical notation, not calculator notation. Do not accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of \( k \), the markscheme will say \( k = 3 \), but the marks will be for the correct value 3 –
there is usually no need for the “k = ”. In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say “must be an equation”.

The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are M marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first A1 is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, FT marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer.
Where numerical answers are required as the final answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value
the exact value if applicable, the correct 3 sf answer
Units will appear in brackets at the end.
Section A

1. (a) valid approach 
\[ eg \ 1.5 - 0.3, 1.5 - 2.7, 2.7 = 0.3 + 2d \]
\[ d = 1.2 \] 
\[ A1 \quad N2 \quad [2 \text{ marks}] \]

(b) correct substitution into term formula 
\[ eg \ 0.3 + 1.2(30 - 1), \ u_{30} = 0.3 + 29(1.2) \]
\[ u_{30} = 35.1 \] 
\[ A1 \quad N2 \quad [2 \text{ marks}] \]

(c) correct substitution into sum formula 
\[ eg \ S_{30} = \frac{30}{2}(0.3 + 35.1), \frac{30}{2}(2(0.3) + 29(1.2)) \]
\[ S_{30} = 531 \] 
\[ A1 \quad N2 \quad [2 \text{ marks}] \]

Total [6 marks]

2. (a) evidence of choosing sine rule 
\[ eg \ \frac{a}{\sin A} = \frac{b}{\sin B} \]
correct substitution 
\[ eg \ \frac{a}{\sin 1.75} = \frac{7}{\sin 0.82} \]
9.42069
BD = 9.42 (cm) 
\[ A1 \quad N2 \quad [3 \text{ marks}] \]

(b) evidence of choosing cosine rule 
\[ eg \ \cos B = \frac{d^2 + c^2 - b^2}{2dc}, \ a^2 = b^3 + c^2 - 2bc \cos B \]
correct substitution 
\[ eg \ \frac{8^2 + 9.42069^2 - 12^2}{2 \times 8 \times 9.42069}, 144 = 64 + BD^2 - 16BD \cos B \]
1.51271
\[ \hat{DBC} = 1.51 \text{ (radians)} \text{ (accept 86.7°)} \] 
\[ A1 \quad N2 \quad [3 \text{ marks}] \]

Total [6 marks]
3. (a) (i) \( y = -1 \)  \( \quad A1 \quad N1 \)

(ii) valid attempt to find \( x \)-intercept  \( \quad (M1) \)

\[ f(x) = 0 \]

\[ 1.38629 \]

\[ x = 2 \ln 2 \text{ (exact), } 1.39 \]  \( A1 \quad N2 \)

(iii) \( y = -2 \) (must be equation)  \( A1 \quad N1 \)

[4 marks]

(b)
4. (a) valid approach  
   \[ h(0) = 2 \text{ (m)} \]  
   \[ h(0) = -15\cos(1.2 \times 0) + 17, -15(1) + 17 \]  
   \[ (M1) \]
   \[ A1 \quad N2 \quad [2 \text{ marks}] \]

   (b) correct substitution into equation  
   \[ 20 = -15\cos1.2t + 17, -15\cos1.2k = 3 \]
   valid attempt to solve for \( k \)  
   \[ \cos1.2k = -\frac{3}{15} \]
   
   1.47679
   \[ k = 1.48 \]
   \[ A1 \quad N2 \quad [3 \text{ marks}] \]

   (c) recognize the need to find the period (seen anywhere)  
   \[ \text{next } t \text{ value when } h = 20 \]
   correct value for period  
   \[ \text{period} = \frac{2\pi}{1.2}, 5.23598, 6.7 - 1.48 \]
   5.2 (min) (must be 1 dp)  
   \[ A1 \quad N2 \quad [3 \text{ marks}] \]
   Total [8 marks]

5. (a) 11 terms  
   \[ (M1) \]
   \[ A1 \quad N1 \quad [1 \text{ mark}] \]

   (b) valid approach  
   \[ (10\choose r)x^{10-r}(\frac{2}{x})^r, a^{10}b^0 + (\frac{10}{1})a^9b^1 + (\frac{10}{2})a^8b^2 + \ldots \]
   Pascal's triangle to 11th row  
   valid attempt to find value of \( r \) which gives term in \( x^8 \)  
   \[ (M1) \]
   \[ A1 \quad N1 \quad [5 \text{ marks}] \]
   Total [6 marks]
6. (a) \[ P(S < 50) = 0.0668 \text{ (accept } P(S \leq 49) = 0.0548 \text{) } \]

\[ A2 \quad N2 \]

[2 marks]

(b) valid approach
\[ Eg \quad P(S < 50) \times P(R < x) \]

correct equation (accept any variable)
\[ eg \quad P(S < 50) \times P(R < x) = 1\%, \quad 0.0668072 \times p = 0.01, \quad P(R < x) = \frac{0.01}{0.0668} \]

finding the value of \( P(R < x) \)
\[ eg \quad \frac{0.01}{0.0668}, \quad 0.149684 \]

\[ x = 9.41 \text{ (accept } x = 9.74 \text{ from } 0.0548) \]

\[ A1 \quad N3 \]

Total [6 marks]

7. correct approach
\[ eg \quad s = \int v, \quad \int_0^p 6t - 6dt \]

correct integration
\[ eg \quad \int 6t - 6dt = 3t^2 - 6t + C, \quad [3t^2 - 6t]_0^p \]

recognizing that there are two possibilities
\[ eg \quad 2 \text{ correct answers, } s = \pm 2, \quad c \pm 2 \]

two correct equations in \( p \)
\[ eg \quad 3p^2 - 6p = 2, \quad 3p^2 - 6p = -2 \]

\[ 0.42265, \quad 1.57735 \]

\[ p = 0.423 \text{ or } p = 1.58 \]

\[ A1A1 \quad N3 \]

[7 marks]
Section B

8. **Note:** There may be slight differences in answers, depending on which values candidates carry through in subsequent parts. Accept answers that are consistent with their working.

(a) (i) valid approach \( (M1) \)
\[ eg \quad \text{correct value for } r \text{ (or for } a \text{ or } b \text{ seen in (ii)) } \]
\[ -0.994347 \]
\[ r = -0.994 \quad A1 \quad N2 \]
(ii) \( -1.58095, \quad 33480.3 \)
\[ a = -1.58, \quad b = 33500 \quad A1A1 \quad N2 \]
\[ \text{[4 marks]} \]

(b) correct substitution into their regression equation \( (A1) \)
\[ eg \quad -1.58095(11000) + 33480.3 \]
\[ 16089.85 \quad (16120 \text{ from } 3\text{sf}) \quad (A1) \]
\[ \text{price} = 16100 \text{ (dollars) (must be rounded to the nearest 100 dollars)} \quad A1 \quad N3 \]
\[ \text{[3 marks]} \]

(c) **METHOD 1**
valid approach \( (M1) \)
\[ eg \quad P \times (\text{rate})' \]
\[ \text{rate} = 0.95 \text{ (may be seen in their expression)} \quad (A1) \]
correct expression \( (A1) \)
\[ eg \quad 16100 \times 0.95^6 \]
\[ 11834.97 \]
\[ 11800 \text{ (dollars)} \quad A1 \quad N2 \]

**METHOD 2**
attempt to find all six terms \( (M1) \)
\[ eg \quad \left( ((16100 \times 0.95) \times 0.95) \ldots \right) \times 0.95, \text{table of values} \]
5 correct values \( (\text{accept values that round correctly to the nearest dollar}) \)
\[ 15295, \quad 14530, \quad 13804, \quad 13114, \quad 12458 \quad A2 \]
\[ 11835 \]
\[ 11800 \text{ (dollars)} \quad A1 \quad N2 \]
\[ \text{[4 marks]} \]

continued…
Question 8 continued

(d) **METHOD 1**

Correct equation

\[ 16100 \times 0.95^t = 10000 \]  

Valid attempt to solve, using logs

\[ 9.28453 \]  

Year 2019

\[ A1 \quad N2 \]

**METHOD 2**

Valid approach using table of values

Both crossover values (accept values that round correctly to the nearest dollar)

\[ P = 10147 \quad (1 \text{ Jan } 2019), \quad P = 9639.7 \quad (1 \text{ Jan } 2020) \]

Year 2019

\[ A1 \quad N2 \]

[4 marks]

Total [15 marks]
9. (a) \( y = 2 \) (correct equation only)

(b) valid approach

\[ \text{eg } (x-1)^{-1} + 2, \quad f'(x) = \frac{0(x-1)-1}{(x-1)^2} \]

\[ -(x-1)^{-2}, \quad f'(x) = \frac{-1}{(x-1)^2} \]

(c) correct equation for the asymptote of \( g \)

\[ \text{eg } y = b \]

\[ b = 2 \]

(d) correct derivative of \( g \) (seen anywhere)

\[ \text{eg } g'(x) = -ae^{-x} \]

correct equation

\[ \text{eg } -c = -ae^{-1} \]

7.38905

\[ a = e^2 \text{ (exact), 7.39} \]

(e) attempt to equate their derivatives

\[ \text{eg } f'(x) = g'(x), \quad \frac{-1}{(x-1)^2} = -ae^{-x} \]

valid attempt to solve their equation

\[ \text{eg } \text{correct value outside the domain of } f \text{ such as } 0.522 \text{ or } 4.51, \]

\includegraphics[width=0.2\textwidth]{diagram.png}

\[ \text{correct solution (may be seen in sketch)} \]

\[ \text{eg } x = 2, \ (2, -1) \]

\[ \text{gradient is } -1 \]

Total [14 marks]
10. (a) valid approach

\[ \text{eg} \quad \vec{B} - \vec{A} , \ \vec{A}O + \vec{OB} , \ \begin{pmatrix} -9 \\ 9 \\ -6 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \\ -7 \end{pmatrix} \]

\[ \vec{AB} = \begin{pmatrix} -10 \\ 4 \\ 1 \end{pmatrix} \]

[2 marks]

(b) valid approach

\[ \text{eg} \quad \vec{OC} = \vec{OA} + \vec{AC} , \ \begin{pmatrix} 1+6 \\ 5-4 \\ -7+0 \end{pmatrix} \]

\[ C(7,1,-7) \]

[2 marks]

(c) any correct equation in the form \[ \vec{r} = a + tb \] (accept any parameter for \( t \))

where \( a \) is \( \begin{pmatrix} -9 \\ 9 \\ -6 \end{pmatrix} \), and \( b \) is a scalar multiple of \( \begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix} \)

\[ \text{eg} \quad \vec{r} = \begin{pmatrix} 9 \\ -4 \\ -6 \end{pmatrix} + t \begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix} \]

\[ \vec{r} = -9i + 9j - 6k + s(6i - 4j + 0k) \]

[2 marks]

(d) correct magnitudes

\[ \text{eg} \quad \sqrt{(-10)^2 + (-4)^2 + 1^2}, \ \sqrt{6^2 + (-4)^2 + (0)^2}, \ \sqrt{10^2 + 4^2 + 1}, \ \sqrt{6^2 + 4^2} \]

\[ k = \frac{\sqrt{117}}{\sqrt{52}} = 1.5 \] (exact)

[3 marks]

continued…
Question 10 continued

(e) correct interpretation of relationship between magnitudes \((A1)\)

\[ \text{eg } AB = 1.5AC, \ BD = 1.5AC, \ \sqrt{117} = \sqrt{52t^2} \]

recognizing D can have two positions (may be seen in working) \(R1\)

\[ \text{eg } \vec{BD} = 1.5 \vec{AC} \text{ and } \vec{BD} = -1.5 \vec{AC}, \ t = \pm 1.5, \ \text{diagram, two answers} \]

valid approach (seen anywhere) \((M1)\)

\[ \text{eg } \vec{OD} = \vec{OB} + \vec{BD}, \ \begin{pmatrix} -9 \\ 9 \\ -6 \end{pmatrix} + t \begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix}, \ BD = k \begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix} \]

one correct expression for \(\vec{OD}\) \((A1)\)

\[ \text{eg } \vec{OD} = \begin{pmatrix} -9 \\ 9 \\ -6 \end{pmatrix} + 1.5 \begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix}, \ \begin{pmatrix} -9 \\ 9 \\ -6 \end{pmatrix} - 1.5 \begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix} \]

\[ D = (0, 3, -6), \ D = (-18, 15, -6) \] (accept position vectors) \(A1A1 \text{ } N3\)

\([6 \text{ marks}]\)

Total [15 marks]