Self-assessment: 12 Basic differentiation and its applications

1. Differentiate the following:
   
   (a) \( \sqrt{x} - \frac{1}{\sqrt{x}} \)
   
   (b) \( \tan x + 2 \cos x \)
   
   (c) \( x^2 - e^x \)
   
   (d) \( 3 \ln x - 1 \)

   \[ 8 \text{ marks} \]

2. Do not use a calculator to answer this question.

   Find the equation of the normal to the curve \( y = 2x - \ln x \) at the point where \( x = 3 \).

   \( (\text{accessible to students on the path to grade 3 or 4}) \) \[ 6 \text{ marks} \]

3. Find the exact coordinates of the stationary point on the graph of \( y = 3e^x - x \).

   \( (\text{accessible to students on the path to grade 3 or 4}) \) \[ 6 \text{ marks} \]

4. (a) (i) Expand and simplify \( (x + h)^2 - x^2 \).

   (ii) Hence prove from first principles that the derivative of \( x^2 \) is \( 2x \).

   \( (\text{accessible to students on the path to grade 5 or 6}) \)

(b) The function \( f \) is defined by \( f(x) = x^2 + 4 \cos x \) for \( 0 < x < \pi \).

   (i) By considering the graphs of \( y = x \) and \( y = 2 \sin x \), show that \( f(x) \) has only one stationary point, and explain why this stationary point is between \( \frac{\pi}{2} \) and \( \pi \).

   (ii) Find \( f''(x) \) and hence prove that the stationary point is a minimum.

   (iii) Find the coordinates of the point of inflection on the graph of \( y = f(x) \).

   (iv) Sketch the graph of \( y = f(x) \), clearly labelling the stationary point and the point of inflection.

   \( (\text{accessible to students on the path to grade 7}) \)

\[ 19 \text{ marks} \]