Statistics and Probability

workbook markscheme

~ categorized past IB Paper 1 and Paper 2 examination questions ~

IB DP Mathematics Standard Level

Topic 5
This workbook contains past Paper 1 and Paper 2 IB examination questions categorized according to major concepts in this topic.

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The use of GDC is not permitted for Paper 1 but is required for Paper 2 questions.
5.1 Measures of Central Tendency

1. (a) 18 A1 N1
   (b) (i) 10 A2 N2
   (ii) 44 A2 N2

2. (a) Mean = \( \frac{\sum fx}{\sum f} \)

\[ \sum fx = (1)(0) + (2)(4) + (3)(6) + (4)(k) + (5)(8) + (6)(k) + (7)(6) \] (A1)

\[ \sum f = k + 30 \] (A1)

Using mean 4.6 = \( \frac{144 + 4k}{k + 30} \) (M1)

4.6k + 138 = 144 + 4k (A1)
0.6k = 6
k = 10 (A1) (C5)

(b) Mode = 4 (A1) (C1)

(accept 5, if k < 8)

3. \( \frac{(10 \times 1) + (20 \times 2) + (30 \times 5) + (40 \times k) + (50 \times 3)}{k + 11} = 34 \) (M1)(A1)

\[ \frac{40k + 350}{k + 11} = 34 \] (A1)
\[ \Rightarrow k = 4 \] (A1) (C4)

4. List of frequencies with \( p \) in the middle

eg 5 + 10, \( p \), 6 + 2 \( \Rightarrow \) 15, 8, or 15 < \( \frac{23 + p}{2} \), or \( p > 7 \) (M1)

Consideration that \( p < 10 \) because 2 is the mode or discretionary for further processing. (M1)

Possible values of \( p \) are 8 and 9 (A2)(A2) (C6)
5.1 Measures of Central Tendency

1. (a) \[ \text{mean} = \frac{\sum x}{n} = \frac{2230}{45} \] (M1)
   \[ \bar{x} = 49.6 \] (Accept 50) (A1) (C2)

(b) \[ \bar{y} = \frac{\sum y}{n+2} \] (may be implied) (M1)
   \[ \sum y = 2230 + 37 + 30 \] (A1)
   \[ \bar{y} = \frac{2297}{47} \] (A1)
   \[ = 48.9 \] (Accept 49) (A1) (C4)

2. Jan–Sept \[ \sum = 630 \times 9 = 5670 \] (M1)(A1)
   Oct–Dec \[ \sum = 810 \times 3 = 2430 \] (M1)(A1)
   \[ \bar{x} = \frac{5670 + 2430}{12} \] (M1)
   \[ \text{mean} = 675 \] (A1) (C6)

3. (a) Median = middle number of 75
   = 38th number
   = 4 (M1)

(b) Mean = \[ \frac{5 + 18 + 48 + 72 + 100 + 42}{75} \] (M1)
   = \[ \frac{285}{75} \]
   = 3.8 (A1) (C2)

4. Mean = \[ \frac{(72 \times 1.79) + (28 \times 1.62)}{100} \] (M1)(M1)(M1)
   = 1.7424 (= 1.74 to 3 sf) (A1) (C4)
5. (a) (i) 10
(ii) \(14 + 10 = 24\)

(b)

\[
\begin{array}{|c|c|}
\hline
x_i & f_i \\
15 & 1 \\
25 & 5 \\
35 & 7 \\
45 & 9 \\
55 & 10 \\
65 & 16 \\
75 & 14 \\
85 & 10 \\
95 & 8 \\
80 & \text{(AG)} \\
\hline
\end{array}
\]

(i) \(\mu = 63\)  
(ii) \(\sigma = 20.5\) (3 sf)

(c) Assymetric diagram/distribution

(d)

\[\text{cumulative frequency} \]
\[
\begin{array}{c|c}
\hline
\text{length (cm)} & \text{cumulative frequency} \\
50 & 20 \\
60 & 40.5 \\
70 & 40.5 \\
80 & \text{(AG)} \\
\hline
\end{array}
\]

Note: This answer assumes appropriate use of a calculator with correct arguments.

OR Median = 65

OR Linear interpolation on the table:

\[
\left(\frac{48 - 40.5}{48 - 32}\right) \times 60 + \left(\frac{40.5 - 32}{48 - 32}\right) \times 70 = 65 \text{ (2sf)}
\]
6. (a) Correct mid interval values 14, 23, 32, 41, 50 (A1)

Substituting into \( \sum \frac{f}{w} \)

\[
\bar{w} = \frac{7(14) + 12(23) + 13(32) + 10(41) + 8(50)}{50}
\]

\( \bar{w} = \frac{1600}{50} \) (A1)

\( \bar{w} = 32 \) (kg) AG N0

(b) Total weight of other boxes = 1600 – 50x (A1)

Total number of other boxes = 50 – x (A1)

Setting up their equation M1

\[
\text{eg } \frac{1600 - 50x}{50 - x} = 30, \ 1600 - 50x = 1500 - 30x
\]

\( x = 5 \) A1 N3

(c) Setting up their inequality M1

Correct substitution A1

\[
\text{eg } \frac{98 + 276 + 416 + 41(10 + y) + 400}{50 + y} < 33, \ \frac{1600 - 41y}{50 + y} < 33
\]

1600 + 41y < 1650 + 33y (A1)

8y < 50 (y < 6.25) A1

6 A1 N1

Note: If candidates don’t use the mid-interval values, but assume that all the new boxes weigh the minimum amount for Class D, award marks as follows:

Setting up their inequality M1

Correct substitution A1

\[
\text{eg } \frac{1600 - 36.5y}{50 + y} < 33
\]

1600 + 36.5y < 1650 + 33y (A1)

3.5y < 50 (y < 14.28...) A1

14 A1 N1

[12]
5.2 Measures of Spread

1. (a) evidence of using $\sum f_i = 100$ (M1)
   \[ k = 4 \]
   A1 N2

   (b) (i) evidence of median position (M1)
   e.g. 50th item, $26 + 10 + 20 = 56$
   median = 3 A1 N2

   (ii) $Q_1 = \text{and} Q_3 = 5$ (A1)(A1)
   interquartile range = 4 (accept 1 to 5 or 5−1, etc.) A1 N3

2. (a) A = 18, B = 19, C = 23, D = 31, E = 36 A1A1A1A1A1 N5

   (b) IQR = 12 A1 N1

3. $b = 3, c = 3$ A1A1 N2

   using mean \( \frac{a+b+c+d}{4} = 4 \) M1

   using range \( (d - a = 6) \)
   \[ a = 2, d = 8 \] A1A1 N2

4. (a) (i) \( r = 10 \) A2 N2

   (ii) \( s = 13 \) A2 N2

   (b) Using $\sum x = 10$
   \[ t = 18 \] A1 N1

5. (a) 3 A1 N1

   (b) 6 A2 N2

   (c) Recognizing the link between 6 and the upper quartile (M1)
   \[ \text{eg 25% scored greater than 6,} \]
   \[ 0.25 \times 32 \]
   \[ 8 \] (A1)
   \[ A1 \] N3

6. \( d = 11; \ c = 11 \) (A1)(A1)(C1)(C1)

   \( d - a = 8 \) (or \( 11 - a = 8 \)) (A1)

   \[ a = 3 \] (A1) (C2)

   \[ \frac{3+b+11+11}{4} = 8 \text{ or } \sum \frac{4}{4} = 8 \] (A1)

   \[ b = 7 \] (A1) (C2)
7. 

<table>
<thead>
<tr>
<th>x</th>
<th>f</th>
<th>Σf</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>7</td>
</tr>
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<td>6</td>
<td>4</td>
<td>11</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>18</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>21</td>
</tr>
</tbody>
</table>

(a) \( m = 6 \) (A2) (C2)
(b) \( Q_1 = 5 \) (A2) (C2)
(c) \( Q_3 = 8 \)

\[
\text{IQR} = 8 - 5 = 3 \quad \text{(accept 5 - 8 or [5, 8])} \quad \text{(C2)}
\]

8. Median = middle value \( \Rightarrow b = 11 \) (A1)
Mean = \( \frac{a + b + c}{3} = \frac{a + 11 + c}{3} = 9 \Rightarrow a + 11 + c = 27 \) (M1)

\( \Rightarrow a + c = 16 \) (A1)
Range = \( c - a = 10 \) (M1)(A1)
Solving equations simultaneously gives \( a = 3 \) (A1) (C6)

9. (a) \( m = \frac{300}{25} = 12 \) (M1) (A1) (C2)
(b) \( s = \sqrt{\frac{625}{25}} = 5 \) (M1) (A1) (C2)
### 5.2 Measures of Spread

**Paper 2**

1. (a) \( \Sigma fx = 1(2) + 2(4) + \ldots + 7(4), \Sigma fx = 146 + 5x \) (seen anywhere) \( \text{A1} \)

   evidence of substituting into mean = \( \frac{\sum fx}{\sum f} \) (M1)

   correct equation \( \text{A1} \)

   e.g. \( \frac{146 + 5x}{34 + x} = 4.5, \) \( 146 + 5x = 4.5(34 + x) \)

   \( x = 14 \) \( \text{A1 N2} \)

   (b) \( \sigma = 1.54 \) \( \text{A2 N2} \)

2. (a) \( \sigma = 1.61 \) \( \text{A2 N2} \)

   (b) median = 4.5 \( \text{A1 N1} \)

   (c) \( Q_1 = 3, Q_3 = 5 \) (may be seen in a box plot) \( \text{(A1)(A1)} \)

   IQR = 2 (accept any notation that suggests the interval 3 to 5) \( \text{A1 N3} \)
1. (a) correct end points
   max = 27, min = 4
   range = 23
   (A1)(A1)

   (b) Graph 3
   A1 N3 3

2. (a) Lines on graph
   100 students score 40 marks or fewer.
   A1 N2

   (b) Identifying 200 and 600
   Lines on graph
   \( a = 55, b = 75 \)
   A1 A1 A1 A1

3. (a) (i) \( m = 165 \)
   (A1)

   (ii) Lower quartile (1st quarter) = 160
       Upper quartile (3rd quarter) = 170
       IQR = 10
       (A1) (A1)

   (b) Recognize the need to use the 40th percentile, or 48th student
       eg a horizontal line through (0, 48)
       \( a = 163 \)
       (M1) A1 N2

4. (a) D B C
   A1 A1 A1 N3

   (b) B A C
   A1 A1 A1 N3
5. (a) Lines on graph (M1)
100 students score 40 marks or fewer. A1 2
(b) Identifying 200 and 600
Lines on graph. (M1)
a = 55, b = 75. A1 4

6. (a) Mark \(x\)
\[\begin{array}{c|c|c|c|c|c}
0 \leq x < 20 & 20 \leq x < 40 & 40 \leq x < 60 & 60 \leq x < 80 & 80 \leq x < 100 \\
\hline
\text{Number of Students} & 22 & 50(\pm 1) & 66(\pm 1) & 42(\pm 1) & 20 \\
\end{array}\]

(b) 40th Percentile \(\Rightarrow\) 80th student fails, (mark 42%) (M2)
Pass mark 43% (Accept mark > 42.) (A1) (C3) [6]

7. (a) Line(s) on graph (M1)
median is 183 (A1) (C2)
(b) Lower quartile \(Q_1 = 175\) (A1)
Upper quartile \(Q_3 = 189\) (A1)
IQR is 14 (Accept 189 – 175, 175 to 189, 189 to 175 and 175 – 189) (M1)(A1) (C4) [6]
(a) (i) Correct lines drawn on graph, median = 20

(ii) Correct lines drawn on graph, UQ = $Q_3 = 24$

(b) $IQR = Q_3 - Q_1 (or UQ - LQ)$

= 10 (accept 14 to 24)
5.3 Cumulative Frequency

1. (a) (i) evidence of appropriate approach
   e.g. $9 + 25 + 35, 34 + 35$
   \[ p = 69 \]
   (M1) A1 N2

   (ii) evidence of valid approach
   e.g. $109 - \text{their value of } p, 120 - (9 + 25 + 35 + 11)$
   \[ q = 40 \]
   (M1) A1 N2

   (b) evidence of appropriate approach
   e.g. substituting into \( \sum \frac{f_i}{n} \), division by 120
   mean = 3.16 (M1) A1 N2

   (c) 1.09 A1 N1

2. (a) (i) \( p = 65 \) A1 N1

   (ii) for evidence of using sum is 125 (or 99 – \( p \))
   \[ q = 34 \]
   (M1) A1 N2

   (b) evidence of median position
   e.g. 63\textsuperscript{rd} student, \( \frac{125}{2} \)
   median is 17 (sit-ups) A1 N2

   (c) evidence of substituting into \( \sum \frac{f(x)}{125} \)
   e.g. \( \frac{15(11) + 16(21) + 17(33) + 18(34) + 19(18) + 20(8)}{125} \) \( \frac{2176}{125} \)
   mean = 17.4 A1 N2

3. (a) median \( m = 32 \) A1 N1

   (b) lower quartile \( Q_1 = 22 \), upper quartile \( Q_3 = 40 \)
   interquartile range = 18 A1 N3

   (c)

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>Number of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 ( \leq t &lt; 10 )</td>
<td>5</td>
</tr>
<tr>
<td>10 ( \leq t &lt; 20 )</td>
<td>11</td>
</tr>
<tr>
<td>20 ( \leq t &lt; 30 )</td>
<td>20</td>
</tr>
<tr>
<td>30 ( \leq t &lt; 40 )</td>
<td>24</td>
</tr>
<tr>
<td>40 ( \leq t &lt; 50 )</td>
<td>14</td>
</tr>
<tr>
<td>50 ( \leq t &lt; 60 )</td>
<td>6</td>
</tr>
</tbody>
</table>

   A1A1 N2 [6]
4. (a) (i) 50 (accept 49, “fewer than 50”)  
(ii) Cumulative frequency (7) = 90  
\[ 90 - 50 = 40 \]  
(iii) 75th or 75.5th person  
median = 6.25 (min), 6 min 15 secs  
(b) Evidence of finding 40% (60%) of 150  
Number spending less than \( k \) minutes is \( (150 - 60) = 90 \)  
\( k = 7 \)  
(c) (i) 
\[ \begin{array}{|c|c|c|c|c|c|c|} \hline 
 t \text{ (minutes)} & 0 \leq t < 2 & 2 \leq t < 4 & 4 \leq t < 6 & 6 \leq t < 8 & 8 \leq t < 10 & 10 \leq t < 12 \\
 \text{Frequency} & 10 & 23 & 37 & 38 & 27 & 15 \\
 \hline 
\end{array} \]  
(ii) Evidence of using all correct mid-interval values (1, 3, 5, 7, 9, 11)  
\[ \text{mean} = \left( \frac{1\times10 + 3\times23 + 5\times37 + 7\times38 + 9\times27 + 11\times15}{150} \right) \]  
\[ = 6.25 \text{ (min), 6 min 15 secs} \]  
[14]

5. (a)
(b) \( Q_1 = 135 \pm 5 \) \( Q_3 = 240 \pm 5 \) \[ \text{(M1)(A1)} \]
Interquartile range = 105 \pm 10. (Accept 135 – 240 or 240 – 135.) \[ \text{(A1)} \]
\[ \text{3} \]

\text{Note: Award (M1) for the correct lines on the graph.}

(c) \( a = 94 – 87 = 7, \ b = 100 – 94 = 6 \) \[ \text{(A1)(A1)} \]

(d) \[ \text{mean} = \frac{12(50) + 46(150) + 29(250) + 7(350) + 6(450)}{100} = 199 \text{ or } \$199000 \] \[ \text{(M1)} \]
OR
\[ \text{mean} = 199 \text{ or } \$199000 \] \[ \text{(A1)} \]

(e) (i) \$350000 => 91.5
Number of \textit{De luxe} houses \( \geq 100 – 91.5 \)
\[ = 9 \text{ or } 8 \] \[ \text{(A1)} \]

(ii) \[ P (\text{both } > 400000) = \frac{6(5)}{9(8)} = \frac{5}{12} \text{ or } \frac{6(5)}{8(7)} = \frac{15}{28} \] \[ \text{(M1)(A1)} \]

6. (a) (i) median fare = \$24 (\pm 0.5) \[ \text{(A1)} \]
(ii) fare \( \leq \$35 \) => number of cabs is 154 (or 153) \[ \text{(A1)} \]

(b) 40% of cabs = 80 cabs
fares up to \$22
distance = \$22 ÷ \$0.55
\[ a = 40 \text{ km} \] \[ \text{(A1)} \]

(c) Distance 90 km => fare = \$90 \times \$0.55
\[ = \$49.50 \] \[ \text{(A1)} \]
Fare \$49.50 => number of cabs = 200 – 186
\[ = 14 \] \[ \text{(A1)} \]
Thus percentage is \[ \frac{14}{200} = 7\% \] \[ \text{(A1)} \]

7. (a) \( s = 7.41(3 \text{ sf}) \) \[ \text{(G3)} \]

(b) \begin{tabular}{|c|c|c|c|c|c|c|}
\hline
Weight \((W)\) & \(W \leq 85\) & \(W \leq 90\) & \(W \leq 95\) & \(W \leq 100\) & \(W \leq 105\) & \(W \leq 110\) & \(W \leq 115\) \\
\hline
Number of packets & 5 & 15 & 30 & 56 & 69 & 76 & 80 \\
\hline
\end{tabular} \[ \text{(A1)} \]

(c) (i) From the graph, the median is approximately 96.8.
Answer: 97 (nearest gram).
\[ \text{(A2)} \]
(ii) From the graph, the upper or third quartile is approximately 101.2.
Answer: 101 (nearest gram).
\[ \text{(A2)} \]

(d) Sum = 0, since the sum of the deviations from the mean is zero.
\[ \text{OR} \]
\[ \sum (W_i - \bar{W}) = \sum W_i - \left(80 \frac{\sum W_i}{80}\right) = 0 \] \[ \text{(M1)(A1)} \]
(c) Let $A$ be the event: $W > 100$, and $B$ the event: $85 < W \leq 110$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \quad \text{(M1)}$$

$$P(A \cap B) = \frac{20}{80} \quad \text{(A1)}$$

$$P(B) = \frac{71}{80} \quad \text{(A1)}$$

$$P(A \mid B) = 0.282 \quad \text{(A1)}$$

**OR**

71 packets with weight $85 < W \leq 110$. (M1)

Of these, 20 packets have weight $W > 100$. (M1)

Required probability $= \frac{20}{71} = 0.282$ (A1)

8. (a) (Using mid-intervals)

$$\bar{v} = \frac{65(7) + 75(25) + ... + 135(5)}{7 + 25 + ... + 5} \quad \text{(M1)}$$

$$= \frac{29450}{300} = 98.2 \text{ km h}^{-1} \quad \text{(A1)}$$

**OR**

$$\bar{v} = 98.2 \quad \text{(G2)}$$

(b) (i) $a = 165, b = 275$ (A1)

(ii)
(c) (i) Vertical line on graph at 105 km h\(^{-1}\)
\[
\frac{300 - 200}{300} \times 100\% = 33.3(\pm 1.3\%)
\]

OR
\[
33.3(\pm 1.3\%)
\]

(ii) 15\% of 300 = 45  \quad 300 - 45 = 255
Horizontal line on graph at 255 cars
Speed = 114(\pm 2 \text{ km h}^{-1})

OR

Speed = 114(\pm 2 \text{ km h}^{-1})

9. (a) \(\bar{x} = \$59\)

OR
\[
\bar{x} = \frac{10 \times 24 + 30 \times 16 + \ldots + 110 \times 10 + 130 \times 4}{24 + 16 + \ldots + 10 + 4}
\]
\[
= \frac{7860}{134}
\]
\[
= \$59
\]

(b)

<table>
<thead>
<tr>
<th>Money ($)</th>
<th>&lt;20</th>
<th>&lt;40</th>
<th>&lt;60</th>
<th>&lt;80</th>
<th>&lt;100</th>
<th>&lt;120</th>
<th>&lt;140</th>
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<tbody>
<tr>
<td>Customers</td>
<td>24</td>
<td>40</td>
<td>62</td>
<td>102</td>
<td>120</td>
<td>130</td>
<td>134</td>
</tr>
</tbody>
</table>
(c) (i) \[ t = 2d^{2/3} + 3 \]
Mean \( d = 59 \) (M1)
Mean \( t \approx 2 \times (59)^{2/3} + 3 \) (A1)
\[ \approx 33.3 \text{ min.} \] (3 sf) (accept 33.2) (A1)

(ii) \[ t > 37 \Rightarrow 2d^{2/3} + 3 > 37 \] (M1)
\[ 2d^{2/3} > 34 \] (A1)
\[ d^{2/3} > 17 \] (A1)
\[ d > (17)^{3/2} \] (A1)
\[ d > 70.1 \] (A1)
From the graph, when \( d = 70.1, \) \( n = 82 \) (A1)
number of shoppers = 134 − 82 = 52 (A1)

10. (a) \[
\begin{array}{c|ccccccccc}
\hline
x & 15 & 45 & 75 & 105 & 135 & 165 & 195 & 225 \\
\hline
f & 5 & 15 & 33 & 21 & 11 & 7 & 5 & 3 \\
\hline
\end{array}
\] (M1)
\[ \bar{x} = 97.2 \text{ (exactly)} \] (A1)

(b) \[
\begin{array}{c|cccccccc}
\hline
x & 30 & 60 & 90 & 120 & 150 & 180 & 210 & 240 \\
\hline
\Sigma f & 5 & 20 & 53 & 74 & 85 & 92 & 97 & 100 \\
\hline
\end{array}
\] (A1)

(c) \[ \text{Median} = 87 \pm 2 \] (A1)
\[ \text{Lower quartile} = 65 \pm 2 \] (A1)
\[ \text{Upper quartile} = 123 \pm 2 \] (A1)
11. (a) 

<table>
<thead>
<tr>
<th>Mark</th>
<th>( \leq 10 )</th>
<th>( \leq 20 )</th>
<th>( \leq 30 )</th>
<th>( \leq 40 )</th>
<th>( \leq 50 )</th>
<th>( \leq 60 )</th>
<th>( \leq 70 )</th>
<th>( \leq 80 )</th>
<th>( \leq 90 )</th>
<th>( \leq 100 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Candidates</td>
<td>15</td>
<td>65</td>
<td><strong>165</strong></td>
<td>335</td>
<td>595</td>
<td>815</td>
<td>905</td>
<td>950</td>
<td>980</td>
<td><strong>1000</strong></td>
</tr>
</tbody>
</table>

(b) 

(c) (i) Median = 46  
(ii) Scores < 35: 240 candidates  
(iii) Top 15\% \( \Rightarrow \) Mark \( \geq 63 \)

12. (a) 76 (mice)  
(b) 11.2 (seconds)  
(c) (i) \( p = 76 - (16 + 22) = 38 \) (allow ft from (ii) (a))  
\( q = 132 - 76 = 56 \)  
(ii) \( x = \frac{7.5 \times 16 + \ldots + 14.5 \times 23}{16 + \ldots + 23} = \frac{3363}{300} \)  
\( = 11.2 \) (accept 11.21)
1. (a) (i) \( n = 0.1 \)
(ii) \( m = 0.2, p = 0.3, q = 0.4 \)

(b) appropriate approach

e.g. \( P(B') = 1 - P(B), m + q, 1 - (n + p) \) (M1)

\[ P(B') = 0.6 \]

2. (a) \( P(A \cap B) = P(A) \times P(B) (= 0.6x) \)

(b) (i) evidence of using \( P(A \cup B) = P(A) + P(B) - P(A)P(B) \) (M1)

correct substitution

e.g. \( 0.80 = 0.6 + x - 0.6x, 0.2 = 0.4x \)

\[ x = 0.5 \]

(ii) \( P(A \cap B) = 0.3 \)

(c) valid reason, with reference to \( P(A \cap B) \) R1

e.g. \( P(A \cap B) \neq 0 \)

3. (a) \( \frac{19}{120} (=0.158) \)

(b) \( 35 - (8 + 5 + 7)(= 15) \) (M1)

Probability = \( \frac{15}{120} = \frac{3}{24} = \frac{1}{8} = 0.125 \)

(c) Number studying = 76 (A1)

Number not studying = 120 - number studying = 44 (M1)

Probability = \( \frac{44}{120} = \frac{11}{30} = 0.367 \)

4. (a)

(b) \( \left( \frac{4}{10} \times \frac{6}{9} \right) + \left( \frac{6}{10} \times \frac{4}{9} \right) \)

\[ = \frac{48}{90} \times \frac{8}{15} = 0.533 \]
5. (a) Independent $\Rightarrow P(A \cap B) = P(A) \times P(B)$  
\[ = 0.3 \times 0.8 \]  
\[ = 0.24 \]  
\[ \text{(M1)} \]  
\[ \text{A1 N2} \]

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$  
\[ = 0.3 + 0.8 - 0.24 \]  
\[ = 0.86 \]  
\[ \text{(M1)} \]  
\[ \text{A1 N1} \]

(c) No, with valid reason  
\[ eg P(A \cap B) \neq 0 \text{ or } P(A \cup B) \neq P(A) + P(B) \text{ or correct numerical equivalent} \]

6. (a) For attempting to use the formula \( P(E \cap F) = P(E)P(F) \)  
\[ \text{Correct substitution or rearranging the formula} \]  
\[ eg \frac{1}{3} = \frac{2}{3} P(F), P(F) = \frac{P(E \cap F)}{P(E)}, P(F) = \frac{1}{3} \times \frac{2}{3} \]
\[ P(F) = \frac{1}{2} \]  
\[ \text{(M1)} \]  
\[ \text{A1 N2} \]

(b) For attempting to use the formula \( P(E \cup F) = P(E) + P(F) - P(E \cap F) \)  
\[ P(E \cup F) = \frac{2}{3} + \frac{1}{2} - \frac{1}{3} \]  
\[ = \frac{5}{6} (= 0.833) \]  
\[ \text{(M1)} \]  
\[ \text{A1 N2} \]

7. Total number of possible outcomes = 36 (may be seen anywhere)  
\[ \text{(A1)} \]

(a) \[ P(E) = P(1,1) + P(2,2) + P(3,3) + P(4,4) + P(5,5) + P(6,6) \]
\[ = \frac{6}{36} \]  
\[ \text{(A1) (C2)} \]

(b) \[ P(F) = P(6,4) + P(5,5) + P(4,6) \]
\[ = \frac{3}{36} \]  
\[ \text{(A1) (C1)} \]

(c) \[ P(E \cup F) = P(E) + P(F) - P(E \cap F) \]
\[ P(E \cap F) = \frac{1}{36} \]  
\[ P(E \cup F) = \frac{6}{36} + \frac{3}{36} - \frac{1}{36} \]
\[ = \frac{8}{36} - \frac{2}{36} = \frac{2}{9} = 0.222 \]  
\[ \text{(M1)(A1) (C3)} \]

[6]
8. Correct probabilities \( \binom{13}{24} \cdot \binom{12}{23} \cdot \binom{11}{22} \cdot \binom{10}{21} \)  

Multiplying \( \binom{13}{24} \cdot \frac{12}{23} \cdot \frac{11}{22} \cdot \frac{10}{21} \)  
\( \text{(M1)} \)

\[ P(4 \text{ girls}) = \frac{17160}{255024} = \frac{65}{966} = 0.0673 \]  
\( \text{(A1) (C6)} \)

9. For using \( P(A \cup B) = P(A) + P(B) - P(A \cap B) \)  
\( \text{(M1)} \)

Let \( P(A) = x \) then \( P(B) = 3x \)

\[ P(A \cap B) = P(A) \times 3P(A) (= 3x^2) \]  
\( \text{(A1)} \)

\[ 0.68 = x + 3x - 3x^2 \]  
\( \text{(A1)} \)

\[ 3x^2 - 4x + 0.68 = 0 \]

\[ x = 0.2 \quad (x = 1.133, \text{ not possible}) \]  
\( \text{(A2)} \)

\[ P(B) = 3x = 0.6 \]  
\( \text{(A1) (C6)} \)

10. \( P(RR) = \frac{7}{12} \times \frac{6}{11} = \frac{7}{22} \)  
\( \text{(M1)(A1)} \)

\( P(YY) = \frac{5}{12} \times \frac{4}{11} = \frac{5}{33} \)  
\( \text{(M1)(A1)} \)

\( P(\text{same colour}) = P(RR) + P(YY) \)  
\( \text{(M1)} \)

\[ = \frac{31}{66} (= 0.470, \text{ 3 sf}) \]  
\( \text{(A1) (C6)} \)

11. (a) \( P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B) \)  
\( \text{(M1)} \)

\[ = \frac{3}{11} + \frac{4}{11} - \frac{6}{11} \]  
\( \text{(M1)} \)

\[ = \frac{1}{11} (0.0909) \]  
\( \text{(A1) (C3)} \)

(b) For independent events, \( P(A \cap B) = P(A) \times P(B) \)  
\( \text{(M1)} \)

\[ = \frac{3}{11} \times \frac{4}{11} \]  
\( \text{(A1)} \)

\[ = \frac{12}{121} (0.0992) \]  
\( \text{(A1) (C3)} \)

12. \( P(\text{different colours}) = 1 - [P(GG) + P(RR) + P(WW)] \)  
\( \text{(M1)} \)

\[ = 1 - \left( \frac{10}{6} \times \frac{9}{25} + \frac{10}{26} \times \frac{9}{25} + \frac{6}{26} \times \frac{5}{25} \right) \]  
\( \text{(A1)} \)

\[ = 1 - \left( \frac{210}{650} \right) \]  
\( \text{(A1)} \)

\[ = \frac{44}{65} (= 0.677, \text{ to 3 sf}) \]  
\( \text{(A1) (C4)} \)
OR

\[ P(\text{different colours}) = P(\text{GR}) + P(\text{RG}) + P(\text{GW}) + P(\text{WG}) + P(\text{RW}) + P(\text{WR}) \]  
\[ = 4 \left( \frac{10}{26} \times \frac{6}{25} \right) + 2 \left( \frac{10}{26} \times \frac{10}{25} \right) \]  
\[ = \frac{44}{65} = 0.677, \text{ to 3 sf} \]

13. (a) \[ U \]

(b) \[ n(A \cup B) = n(A) + n(B) - n(A \cap B) \]
\[ 65 = 30 + 50 - n(A \cap B) \]
\[ \Rightarrow n(A \cap B) = 15 \text{ (may be on the diagram)} \]  
\[ n(B \cap A') = 50 - 15 = 35 \]  
\[ n(U) = 100 \]

(c) \[ P(B \cap A') = \frac{n(B \cap A')}{n(U)} = \frac{35}{100} = 0.35 \]

14. (a)

(b) \[ P(\text{one or more sixes}) = \frac{1}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{5}{6} \times \frac{1}{6} \times \frac{5}{6} \times \frac{1}{6} \]
\[ = \frac{11}{36} \]
15. (a) [Diagram of overlapping circles]

(b) (i) \( n(A \cap B) = 2 \)  

(ii) \( P(A \cap B) = \frac{2}{36} \) or \( \frac{1}{18} \) (allow from (b)(i))  

(c) \( n(A \cap B) \neq 0 \) (or equivalent)  

16. (a) \( p(A \cap B) = 0.6 + 0.8 - 1 = 0.4 \)  

(b) \( p(\overline{A} \cup \overline{B}) = p(\overline{A \cap B}) = 1 - 0.4 = 0.6 \)
5.4 Simple Probability

1. (a) evidence of valid approach involving $A$ and $B$ (M1)  
   
   $e.g.$ $P(A \cap \text{pass}) + P(B \cap \text{pass})$, tree diagram  
   correct expression (A1)  
   
   $e.g.$ $P(\text{pass}) = 0.6 \times 0.8 + 0.4 \times 0.9$  
   $P(\text{pass}) = 0.84$ A1 N2 3

   (b) evidence of recognizing complement (seen anywhere) (M1)  
   
   $e.g.$ $P(B) = x, P(A) = 1 - x, 100 - x, x + y = 1$  
   
   evidence of valid approach (M1)  
   correct expression A1  
   
   $e.g.$ $0.8(1 - x) + 0.9x, 0.8x + 0.9y$  
   
   $0.87 = 0.8(1 - x) + 0.9x, 0.8 \times 0.3 + 0.9 \times 0.7 = 0.87, 0.8x + 0.9y = 0.87$  
   $70\%$ from B A1 N2 4 [7]

2. METHOD 1

   for independence $P(A \cap B) = P(A) \times P(B)$ (R1)  
   
   expression for $P(A \cap B)$, indicating $P(B) = 2P(A)$ (A1)  
   
   $e.g.$ $P(A) \times 2P(A), x \times 2x$  
   
   substituting into $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (M1)  
   correct substitution A1  
   
   $e.g.$ $0.52 = x + 2x - 2x^2, 0.52 = P(A) + 2P(A) - 2P(A)P(A)$  
   
   correct solutions to the equation (A2)  
   $e.g.$ $0.2, 1.3$ (accept the single answer $0.2$)  
   $P(B) = 0.4$ A1 N6

METHOD 2

   for independence $P(A \cap B) = P(A) \times P(B)$ (R1)  
   
   expression for $P(A \cap B)$, indicating $P(A) = \frac{1}{2} P(B)$ (A1)  
   
   $e.g.$ $P(B) \times \frac{1}{2} P(B), x \times \frac{1}{2} x$  
   
   substituting into $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (M1)  
   correct substitution A1  
   
   $e.g.$ $0.5x + x - 0.5x^2, 0.52 = 0.5P(B) + P(B) - 0.5P(B)P(B)$  
   
   correct solutions to the equation (A2)  
   $e.g.$ $0.4, 2.6$ (accept the single answer $0.4$)  
   $P(B) = 0.4$ (accept $x = 0.4$ if $x$ set up as $P(B)$) A1 N6 [7]
3. (a) evidence of binomial distribution (seen anywhere) 
\( e.g. X \sim B \left( 3, \frac{1}{4} \right) \)
\[
\text{mean} = \frac{3}{4} (= 0.75) \quad \text{A1 N2}
\]

(b) \( P(X = 2) = \binom{3}{2} \left( \frac{1}{4} \right)^2 \left( \frac{3}{4} \right) \) \( \text{(A1)} \)
\[
P(X = 2) = 0.141 \quad \left( \frac{9}{64} \right) \quad \text{A1 N2}
\]

(c) evidence of appropriate approach 
\( e.g. \) complement, \( 1 - P(X = 0) \), adding probabilities
\[
P(X = 0) = (0.75)^3 = \left( 0.422, \frac{27}{64} \right) \quad \text{(A1)}
\]
\[
P(X \geq 1) = 0.578 \quad \left( \frac{37}{64} \right) \quad \text{A1 N2}
\]

4. (a) 
\[
\begin{align*}
& R \\
& \quad \frac{2}{3} \\
& \quad \frac{1}{2} \\
& \quad \frac{4}{5} \\
& S
\end{align*}
\]
\[
\begin{align*}
& S' \\
& \quad \frac{1}{2} \\
& \quad \frac{3}{4} \\
& \quad \frac{4}{5} \\
& R'
\end{align*}
\]

(b) (i) \( P(R \cap S) = \frac{1}{3} \times \frac{4}{5} = \frac{4}{15} = 0.267 \) \( \text{(A1) (N1)} \)

(ii) \( P(S) = \frac{1}{3} \times \frac{4}{5} + \frac{2}{3} \times \frac{1}{4} \) \( \text{(A1)(A1)} \)
\[
= \frac{13}{30} (= 0.433) \quad \text{(A1) (N3)}
\]

(iii) \( P(R \mid S) = \frac{4}{15} \) \( \frac{13}{30} \)
\[
= \frac{8}{13} (= 0.615) \quad \text{(A1) (N3)}
\]

[7]

[10]
5.5 Conditional Probability

### 1.

(a) (i) \( s = 1 \)  
A1 N1

(ii) evidence of appropriate approach  
\( e.g. \ 21-16, 12 + 8 - q = 15 \)  
\( q = 5 \)  
A1 N2

(iii) \( p = 7, r = 3 \)  
A1A1 N2 5

(b) (i) \( \frac{5}{8} \)  
A2 N2

(ii) METHOD 1  
\[ P(\text{art}) = \frac{12}{16} = \frac{3}{4} \]  
A1  
evidence of correct reasoning  
R1  
e.g. \( \frac{3}{4} \neq \frac{5}{8} \)  
AG N0

METHOD 2  
\[ P(\text{art}) \times P(\text{music}) = \frac{96}{256} = \frac{3}{8} \]  
A1  
evidence of correct reasoning  
R1  
e.g. \( \frac{12}{16} \times \frac{8}{16} \neq \frac{5}{16} \)  
AG N0 4

(c) \( P(\text{first takes only music}) = \frac{3}{16} = \) (seen anywhere)  
A1  
\( P(\text{second takes only art}) = \frac{7}{15} = \) (seen anywhere)  
A1  
evidence of valid approach  
(M1)  
e.g. \( \frac{3}{16} \times \frac{7}{15} \)  
A1 N2 4

\[ P(\text{music and art}) = \frac{21}{240} = \frac{7}{80} \]

### 2.

(a) (i) \( p = 0.2 \)  
A1 N1

(ii) \( q = 0.4 \)  
A1 N1

(iii) \( r = 0.1 \)  
A1 N1

(b) \( \frac{2}{3} \)  
A2 N2

(c) valid reason  
R1  
e.g. \( \frac{2}{3} \neq 0.5, 0.35 \neq 0.3 \)  
AG N0

\[ \frac{2}{3} \neq 0.5, 0.35 \neq 0.3 \]  
thus, \( A \) and \( B \) are not independent
3. (a) \( p = \frac{4}{5} \)  

(b) multiplying along the branches (M1)  
\( e.g. \frac{1}{5} \times 1 = \frac{12}{40} \)
adding products of probabilities of two mutually exclusive paths (M1)  
\( e.g. \frac{1}{5} \times \frac{1}{4} + \frac{4}{5} \times \frac{3}{8} + \frac{1}{4} + \frac{12}{40} \)  
\( P(B) = \frac{14}{40} \left( = \frac{7}{20} \right) \)  

(c) appropriate approach which must include \( A' \) (may be seen on diagram) (M1)  
\( e.g. \frac{P(A' \cap B)}{P(B)} \left( \text{do not accept} \frac{P(A \cap B)}{P(B)} \right) \)  
\( \frac{4}{5} \times \frac{3}{8} \)  
\( P(A' \mid B) = \frac{5}{7} \)  
\( \frac{20}{7} \)  
\( P(A' \mid B) = \frac{12}{14} \left( = \frac{6}{7} \right) \)  

4. (a) \( P(A) = \frac{1}{11} \)  

(b) \( P(B \mid A) = \frac{2}{10} \)  

(c) recognising that \( P(A \cap B) = P(A) \times P(B \mid A) \) (M1)  
correct values (A1)  
\( e.g. P(A \cap B) = \frac{11}{11} \times \frac{2}{10} \)  
\( P(A \cap B) = \frac{2}{110} \)  

5. (a) \( \frac{3}{4} \)  

(b) \( P(A \cup B) = P(A) + P(B) - P(A \cap B) \) (M1)  
\( P(A \cap B) = P(A) + P(B) - P(A \cup B) \)  
\( = \frac{2}{5} + \frac{3}{4} - \frac{7}{8} \)  
\( = \frac{11}{40} \left( = 0.275 \right) \)  

(c) \( P(A \mid B) = \frac{P(A \cap B)}{P(B)} \left( = \frac{11}{40} \times \frac{3}{4} \right) \)  
\( = \frac{11}{30} \left( = 0.367 \right) \)
6. (a) (i) evidence of substituting into \( n(A \cup B) = n(A) + n(B) - n(A \cap B) \) (M1)
\[ 75 + 55 - 100, \text{ Venn diagram} \]
\[ 30 \]  
A1 N2

(ii) 45  
A1 N1

(b) (i) **METHOD 1**

evidence of using complement, Venn diagram (M1)
\[ \frac{70}{100} = \frac{7}{10} \]
A1 N2

**METHOD 2**

attempt to find \( P(\text{only one sport}) \), Venn diagram (M1)
\[ \frac{25}{100} + \frac{45}{100} \]
\[ \frac{70}{100} = \frac{7}{10} \]  
A1 N2

(ii)  \[ \frac{45}{70} = \frac{9}{14} \]  
A2 N2

(c) valid reason in words or symbols (R1)
\[ e.g. \ P(A \cap B) = 0 \text{ if mutually exclusive}, P(A \cap B) \text{ if not mutually exclusive} \]
correct statement in words or symbols  
A1 N2
\[ e.g. \ P(A \cap B) = 0.3, P(A \cup B) \neq P(A) + P(B), P(A) + P(B) > 1, \text{ some students play both sports, sets intersect} \]

d) valid reason for independence (R1)
\[ e.g. \ P(A \cap B) = P(A) \times P(B), P(B \mid A) = P(B) \]
correct substitution  
A1 A1 N3

7. (a) (i) correct calculation (A1)
\[ e.g. \ \frac{9}{20} + \frac{5}{20} - \frac{2}{20}, \frac{4+2+3+3}{20} \]
\[ P(\text{male or tennis}) = \frac{12}{20} = \frac{3}{5} \]  
A1 N2

(ii) correct calculation (A1)
\[ e.g. \ \frac{6}{20} + \frac{11}{20}, \frac{3+3}{11} \]
\[ P(\text{not football} \mid \text{female}) = \frac{6}{11} \]  
A1 N2

(b) \[ P(\text{first not football}) = \frac{11}{20}, P(\text{second not football}) = \frac{10}{19} \]
P(\text{neither football}) = \[ \frac{11}{20} \times \frac{10}{19} \]  
A1

\[ P(\text{neither football}) = \frac{110}{380} = \frac{11}{38} \]  
A1 N1
(a)  
(i) number of ways of getting \(X = 6\) is 5 \(\quad\text{(A1)}\)

\[
P(X = 6) = \frac{5}{36} \quad \text{A1 N2}
\]

(ii) number of ways of getting \(X > 6\) is 21 \(\quad\text{(A1)}\)

\[
P(X > 6) = \frac{21}{36} = \frac{7}{12} \quad \text{A1 N2}
\]

(iii) \(P(X=7|X>5) = \frac{6}{26} = \frac{3}{13}\) \(\quad\text{(A2 N2)}\)

(b) evidence of substituting into \(E(X)\) formula \(\quad\text{(M1)}\)

finding \(P(X < 6) = \frac{10}{36}\) (seen anywhere) \(\quad\text{(A2)}\)

evidence of using \(E(W) = 0\) \(\quad\text{(M1)}\)

correct substitution \(\quad\text{A2}\)

e.g. \(3 \left(\frac{5}{36}\right) + 1 \left(\frac{21}{36}\right) - k \left(\frac{10}{36}\right) = 0\), \(15 + 21 - 10k = 0\)

\[k = \frac{36}{10} = (3.6) \quad \text{A1 N4}\]

9.  
(a) \(\frac{46}{97} = (0.474)\) \(\quad\text{A1A1 N2}\)

(b) \(\frac{13}{51} = (0.255)\) \(\quad\text{A1A1 N2}\)

(c) \(\frac{59}{97} = (0.608)\) \(\quad\text{A2 N2}\)

10.  
(a) \(P(P|C) = \frac{20}{20+40}\) \(\quad\text{A1}\)

\[
= \frac{1}{3} \quad \text{A1 N1}
\]

(b) \(P(P|C^c) = \frac{30}{30+60}\) \(\quad\text{A1}\)

\[
= \frac{1}{3} \quad \text{A1 N1}
\]

(c) Investigating conditions, or some relevant calculations \(\quad\text{(M1)}\)

\(P\) is independent of \(C\), with valid reason \(\quad\text{A1 N2}\)

e.g. \(P(P|C) = P(P|C^c), P(P|C) = P(P),\)

\[
\frac{20}{150} = \frac{50}{150} \times \frac{60}{150} \quad \text{(ie } P(P \cap C) = P(P) \times P(C))
\]

\[\text{[6]}\]
11. (a) \[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]
\[ P(A \cap B) = \frac{1}{2} + \frac{3}{4} - \frac{7}{8} \]
\[ = \frac{3}{8} \] (M1) (A1) (C2)

(b) \[ P(A \mid B) = \frac{P(A \cap B)}{P(B)} \]
\[ = \frac{\frac{3}{8}}{\frac{3}{4}} \]
\[ = \frac{1}{2} \] (M1) (A1) (C2)

(c) Yes, the events are independent (A1) (C1)
\[ P(A \cap B) = P(A)P(B) \] (R1) (C1)

12. (a) \[ \frac{120}{360} = \frac{1}{3} = 0.333 \] (A1)(A1) (C2)

(b) \[ \frac{90 + 120}{360} = \frac{210}{360} = \frac{7}{12} = 0.583 \] (A2) (C2)

(c) \[ \frac{90}{210} = \frac{3}{7} = 0.429 \] (Accept \( \frac{1}{2} \)) (A1)(A1) (C2)

13. (a) Independent (I) (C2)
(b) Mutually exclusive (M) (C2)
(c) Neither (N) (C2)

14. (a) \[ P = \frac{22}{23} = 0.957 \text{ (3 sf)} \] (A2) (C2)

(b)

OR
\[ P = P(\text{RRG}) + P(\text{RGR}) + P(\text{GRR}) \]
\[ \frac{22 \times 21 \times 3}{25 \times 24 \times 23} + \frac{22 \times 3 \times 21}{25 \times 24 \times 23} + \frac{3 \times 22 \times 21}{25 \times 24 \times 23} \]
\[ = \frac{693}{2300} = 0.301 \text{ (3 sf)} \] (A1) (C4)
15. (a) 

\[ \begin{array}{ccc}
\text{ } & 0.6 & \\
0.4 & \text{ } & 0.4 \\
0.6 & \text{ } & 0.5 \\
\end{array} \]

(b) \[ P(B) = 0.4(0.6) + 0.6(0.5) = 0.24 + 0.30 = 0.54 \] (A1) (C1)

(c) \[ P(C | B) = \frac{P(B \cap C)}{P(B)} = \frac{0.24}{0.54} = \frac{4}{9} = 0.444 \text{ (3 sf)} \] (A1) (C1) [4]

16. (a) 

<table>
<thead>
<tr>
<th></th>
<th>Males</th>
<th>Females</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployed</td>
<td>20</td>
<td>40</td>
<td>60</td>
</tr>
<tr>
<td>Employed</td>
<td>90</td>
<td>50</td>
<td>140</td>
</tr>
<tr>
<td>Totals</td>
<td>110</td>
<td>90</td>
<td>200</td>
</tr>
</tbody>
</table>

(b) (i) \[ P(\text{unemployed female}) = \frac{40}{200} = \frac{1}{5} \] (A1)

(ii) \[ P(\text{male I employed person}) = \frac{90}{140} = \frac{9}{14} \] (A1) [4]

17. (a) 

<table>
<thead>
<tr>
<th></th>
<th>Boy</th>
<th>Girl</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>TV</td>
<td>13</td>
<td>25</td>
<td>38</td>
</tr>
<tr>
<td>Sport</td>
<td>33</td>
<td>29</td>
<td>62</td>
</tr>
<tr>
<td>Total</td>
<td>46</td>
<td>54</td>
<td>100</td>
</tr>
</tbody>
</table>

\[ P(\text{TV}) = \frac{38}{100} \] (A1) (C2) [4]

(b) \[ P(\text{TV} | \text{Boy}) = \frac{13}{46} = 0.283 \text{ (3 sf)} \] (A2) (C2) [4]
### 5.5 Conditional Probability

#### 1.

(a) appropriate approach (M1)

*e.g. tree diagram or a table*

\[
P(\text{win}) = P(H \cap W) + P(A \cap W)
\]

\[
= (0.65)(0.83) + (0.35)(0.26)
\]

\[
= 0.6305 \text{ (or 0.631)} \quad \text{A1} \quad \text{N2}
\]

(b) evidence of using complement (M1)

*e.g. \(1 - p, 0.3695\)

choosing a formula for conditional probability (M1)

*e.g. \(P(H | W') = \frac{P(W' \cap H)}{P(W')}\)

*correct substitution (M1)*

\[
\frac{(0.65)(0.17)}{0.3695} = \frac{0.1105}{0.3695} \quad \text{A1}
\]

\[P(\text{home}) = 0.299 \quad \text{A1} \quad \text{N3}\]

#### 2.

Sample space = \{(1, 1), (1, 2), ..., (6, 5), (6, 6)\} (This may be indicated in other ways, for example, a grid or a tree diagram, partly or fully completed)

\[
P(S < 8) = \frac{6 + 5 + 4 + 3 + 2 + 1}{36} = \frac{7}{12} \quad \text{(M1)}
\]

**OR**

\[P(S < 8) = \frac{7}{12} \quad \text{(A2)}
\]

(b) \(P(\text{at least one 3}) = \frac{1+1+6+1+1+1}{36} = \frac{11}{36} \quad \text{(M1)}\)

**OR**

\[P(\text{at least one 3}) = \frac{11}{36} \quad \text{(A2)}\]
(c) \( P(\text{at least one } 3 \mid S < 8) = \frac{P(\text{at least one } 3 \cap S < 8)}{P(S < 8)} \) (M1)
\[
= \frac{7/36}{7/12} = \frac{1}{3}
\] (A1)

3. (a) \( P(F \cup S) = 1 - 0.14 = 0.86 \) (A1)

Choosing an appropriate formula (M1)

\( \text{eg } P(A \cup B) = P(A) + P(B) - P(A \cap B) \)

Correct substitution

\( \text{eg } P(F \cap S) = 0.93 - 0.86 \)
\( P(F \cap S) = 0.07 \) AG N0

(b) Using conditional probability (M1)

\( \text{eg } P(F \mid S) = \frac{P(F \cap S)}{P(S)} \)
\( P(F \mid S) = \frac{0.07}{0.62} = 0.113 \) A1 N3

(c) \( F \) and \( S \) are not independent A1 N1

EITHER

If independent \( P(F \mid S) = P(F) \), 0.113 \( \neq \) 0.31 R1R1 N2

OR

If independent \( P(F \cap S) = P(F)P(S) \), 0.07 \( \neq \) 0.31 \times 0.62 (= 0.1922) R1R1 N2

(d) Let \( P(F) = x \)
\( P(S) = 2P(F) (= 2x) \) (A1)

For independence \( P(F \cap S) = P(F)P(S) (= 2x^2) \) (R1)

Attempt to set up a quadratic equation (M1)

\( \text{eg } P(F \cup S) = P(F)P(S) - P(F)P(S), 0.86 = x + 2x - 2x^2 \)
\( 2x^2 - 3x + 0.86 = 0 \) A2
\( x = 0.386, x = 1.11 \) (A1)
\( P(F) = 0.386 \) (A1) N5
4. (a) 

(b) (i) \( P(M \text{ and } G) = \frac{1}{3} \times \frac{2}{5} = \frac{2}{15} = 0.133 \)  

(ii) \[ P(G) = \frac{1}{3} \times \frac{2}{5} + \frac{2}{3} \times \frac{8}{10} = \frac{10}{15} = \frac{2}{3} = 0.667 \]  

(iii) \[ P(M | G) = \frac{P(M \cap G)}{P(G)} = \frac{\frac{2}{15}}{\frac{2}{3}} = \frac{1}{5} \text{ or } 0.2 \]

(c) \( P(R) = 1 - \frac{2}{3} = \frac{1}{3} \)

Evidence of using a correct formula: \( M1 \)

\[ E(\text{win}) = 2 \times \frac{1}{3} + 5 \times \frac{2}{3} \left( \frac{3}{5} + 2 \times \frac{2}{3} \times \frac{2}{10} + 5 \times \frac{2}{3} \times \frac{2}{3} + 5 \times \frac{2}{3} \times \frac{8}{10} \right) \]

\[ = \$4 \left( \frac{12}{3}, \frac{60}{15} \right) \]
5. (a) (i) \( P(A) = \frac{80}{210} = \left( \frac{8}{21} = 0.381 \right) \) (A1) (N1)

(ii) \( P(\text{year 2 art}) = \frac{35}{210} = \left( \frac{1}{6} = 0.167 \right) \) (A1) (N1)

(iii) No (the events are not independent, or, they are dependent) (A1) (N1)

EITHER

\[ P(A \cap B) = P(A) \times P(B) \] (to be independent) (M1)

\[ P(B) = \frac{100}{210} \left( \frac{10}{21} = 0.476 \right) \] (A1)

\[ \frac{1}{6} \neq \frac{8}{21} \times \frac{10}{21} \] (A1)

OR

\[ P(A) = P(A \mid B) \] (to be independent) (M1)

\[ P(A \mid B) = \frac{35}{100} \] (A1)

\[ \frac{8}{21} \neq \frac{35}{100} \] (A1)

OR

\[ P(B) = P(B \mid A) \] (to be independent) (M1)

\[ P(B) = \frac{100}{210} \left( \frac{10}{21} = 0.476 \right), P(B \mid A) = \frac{35}{80} \] (A1)

\[ \frac{35}{80} \neq \frac{100}{210} \] (A1)

Note: Award the first (M1) only for a mathematical interpretation of independence.

(b) \( n(\text{history}) = 85 \) (A1)

\[ P(\text{year 1} \mid \text{history}) = \frac{50}{85} = \left( \frac{10}{17} = 0.588 \right) \] (A1) (N2)

2

(c) \[ 2 \left( \frac{110}{210} \times \frac{100}{209} \right) + \left( \frac{100}{210} \times \frac{110}{209} \right) = 2 \times \frac{110}{210} \times \frac{100}{209} \] (M1)(A1)(A1)

\[ = \frac{200}{399} \] (A1)

\[ = 0.501 \] (N2)

4

[12]
6. (a) 

Note: Award (A1) for the given probabilities in the correct positions, and (A1) for each bold value.

(b) Probability that Dumisani will be late is \( \frac{7}{8} \times \frac{1}{4} + \frac{1}{8} \times \frac{3}{5} \) (A1)(A1)

\[ = \frac{47}{160} \] (A1) (N2) 3

(c) \[ P(W|L) = \frac{P(W \cap L)}{P(L)} \]

\[ P(W \cap L) = \frac{7}{8} \times \frac{1}{4} \] (A1)

\[ P(L) = \frac{47}{160} \] (A1)

\[ P(W|L) = \frac{\frac{32}{47}}{\frac{160}{47}} \] (M1)

\[ = \frac{35}{47} = 0.745 \] (A1) (N3) 4

[11]
7. (a)  

\[ \begin{array}{c}
0.9 \\
\text{Red} \\
0.4 \\
\text{Yellow} \\
0.6 \\
\text{Does not grow} \\
0.8 \\
\text{Grows} \\
0.1 \\
\text{Does not grow} \\
0.2 \\
\end{array} \]

(b) (i) \[ 0.4 \times 0.9 = 0.36 \text{ (A1)} \]

(ii) \[ 0.36 + 0.6 \times 0.8 = 0.36 + 0.48 = 0.84 \text{ (A1)} \]

(iii) \[ \frac{\text{P(red ∩ grows)}}{\text{P(grows)}} = \frac{0.36}{0.84} = \frac{3}{7} \text{ (A1)} \]

8. (a)  

\[ n(E \cup H) = a + b + c = 88 - 39 = 49 \]  (M1)

\[ n(E \cup H) = 32 + 28 - b = 49 \]

\[ 60 - 49 = b = 11 \]  (A1)

\[ a = 32 - 11 = 21 \]  (A1)

\[ c = 28 - 11 = 17 \]  (A1)
(b) (i) \[ P(E \cap H) = \frac{11}{88} = \frac{1}{8} \] 

(ii) \[ P(H'|E) = \frac{P(H' \cap E)}{P(E)} = \frac{21}{88} \times \frac{32}{88} \]

\[ = \frac{21}{32} \approx 0.656 \] 

**OR**

Required probability = \[ \frac{21}{32} \] 

(c) (i) \[ P(\text{none in economics}) = \frac{56 \times 55 \times 54}{88 \times 87 \times 86} \]

\[ = 0.253 \] 

(ii) \[ P(\text{at least one}) = 1 - 0.253 \]

\[ = 0.747 \] 

**OR**

\[ 3 \left( \frac{32 \times 56 \times 55}{88 \times 87 \times 86} \right) + 3 \left( \frac{32 \times 31 \times 56}{88 \times 87 \times 86} \right) + \frac{32 \times 31 \times 30}{88 \times 87 \times 86} \]

\[ = 0.747 \]
5.6 Discrete Probability Distributions

1. (a) (i) \( \frac{7}{24} \)  

(ii) evidence of **multiplying** along the branches  
\[ \frac{2}{3} \times \frac{5}{8} \times \frac{1}{3} \times \frac{7}{8} \]  
**adding** probabilities of two mutually exclusive paths  
\[ \frac{1}{3} \times \frac{7}{8} + \frac{2}{3} \times \frac{3}{8} + \frac{1}{3} \times \frac{1}{8} + \frac{2}{3} \times \frac{5}{8} \]  
\[ P(F) = \frac{13}{24} \]

(b) (i) \( \frac{1}{3} \times \frac{1}{8} \)

(ii) recognizing this is \( P(E \mid F) \)
\[ \frac{7}{3} \div \frac{13}{24} \]
\[ \frac{168}{24} \times \frac{7}{13} \]
\[ \frac{312}{13} \]

(c)
\[
\begin{array}{|c|c|c|}
\hline
X (cost in euros) & 0 & 3 & 6 \\
\hline
P (X) & \frac{1}{9} & \frac{4}{9} & \frac{4}{9} \\
\hline
\end{array}
\]

(d) correct substitution into \( E(X) \) formula
\[ \frac{0 \times \frac{1}{9} + 3 \times \frac{4}{9} + 6 \times \frac{4}{9} + \frac{12}{9} + \frac{24}{9} }{24} \]
\[ E(X) = 4 \] (euros)

2. (a)
\[
\begin{array}{ccc}
3, 9 & 4, 9 & 5, 9 \\
3, 10 & 4, 10 & 5, 10 \\
3, 10 & 4, 10 & 5, 10 \\
\end{array}
\]

(b) 12, 13, 14, 15 (accept 12, 13, 13, 13, 14, 14, 14, 15, 15)

(c) \[ P(12) = \frac{1}{9}, P(13) = \frac{3}{9}, P(14) = \frac{3}{9}, P(15) = \frac{2}{9} \]

(d) correct substitution into formula for \( E(S) \)
\[ \frac{12 \times \frac{1}{9} + 13 \times \frac{3}{9} + 14 \times \frac{3}{9} + 15 \times \frac{2}{9}}{9} \]
\[ E(S) = \frac{123}{9} \]
(c) **METHOD 1**

Correct expression for expected gain $E(A)$ for 1 game

*e.g.* $\frac{4}{9} \times 50 - \frac{5}{9} \times 30$

$$E(A) = \frac{50}{9}$$

Amount at end = expected gain for 1 game $\times 36$

$= 200$ (dollars)

**METHOD 2**

Attempt to find expected number of wins and losses

*e.g.* $\frac{4}{5} \times 36, \frac{5}{9} \times 36$

Attempt to find expected gain $E(G)$

*e.g.* $16 \times 50 - 30 \times 20$

$$E(G) = 200$$ (dollars)

3. (a) (i) $P(B) = \frac{3}{4}$

(ii) $P(R) = \frac{1}{4}$

(b) $p = \frac{3}{4}$

$s = \frac{1}{4}, t = \frac{3}{4}$

(c) (i) $P(X = 3)$

$= P(\text{getting 1 and 2}) = \frac{1}{4} \times \frac{3}{4}$

$= \frac{3}{16}$

(ii) $P(X = 2) = \frac{1}{4} \times \frac{1}{4} + \frac{3}{4} \left(1 - \frac{3}{16}\right)$

$= \frac{13}{16}$

(d) (i)

<table>
<thead>
<tr>
<th>$X$</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td>$\frac{13}{16}$</td>
<td>$\frac{3}{16}$</td>
</tr>
</tbody>
</table>

A2 N2
(ii) evidence of using $E(X) = \sum xP(X = x)$

$$E(X) = 2 \frac{13}{16} + 3 \frac{3}{16}$$

$$= \frac{35}{16} = 2 \frac{3}{16}$$

(A1)  

A1  N2

(e) win $10 \Rightarrow$ scores 3 one time, 2 other time

$$P(3) \times P(2) = \frac{13}{16} \times \frac{3}{16}$$

(seen anywhere)  

A1

evidence of recognizing there are different ways of winning $10$

$$\text{e.g. } P(3) \times P(2) + P(2) \times P(3), \ 2 \left( \frac{13}{16} \times \frac{3}{16} \right),$$

$$\frac{36}{256} + \frac{3}{256} + \frac{36}{256} + \frac{3}{256}$$

$$P(\text{win } 10) = \frac{78}{256} = \frac{39}{128}$$

M1

A1  N3

4. (a) $P(X = 2) = \frac{4}{14} \left( = \frac{2}{7} \right)$

A1  N1  1

(b) $P(X = 1) = \frac{1}{14}$

(A1)

$$P(X = k) = \frac{k^2}{14}$$

(A1)

setting the sum of probabilities = 1

M1

e.g. $\frac{1}{14} + \frac{4}{14} + \frac{k^2}{14} = 1, \ 5 + k^2 = 14$

$$k^2 = 9 \left( \text{accept } \frac{k^2}{14} = \frac{9}{14} \right)$$

A1

$$k = 3$$

AG  N0  4

(c) correct substitution into $E(X) = \sum xP(X = x)$

A1

e.g. $1 \left( \frac{1}{14} \right) + 2 \left( \frac{4}{14} \right) + 3 \left( \frac{9}{14} \right)$

$$E(X) = \frac{36}{14} \left( = \frac{18}{7} \right)$$

A1  N1  2

[7]
5. (a) For summing to 1
\[ \frac{1}{5} + \frac{2}{5} + \frac{1}{10} + x = 1 \]
\[ x = \frac{3}{10} \]  
A1 N2

(b) For evidence of using \( E(X) = \sum xf(x) \)  
Correct calculation  
\[ 0.1 \times 1 + 2 \times \frac{2}{5} + 3 \times \frac{1}{10} + 4 \times \frac{3}{10} \]
\[ E(X) = \frac{25}{10} = 2.5 \]  
A1 N2

(c) \[ \frac{1}{10} \times \frac{1}{10} \]  
\[ \frac{1}{100} \]  
A1 N2

6. (a) For summing to 1  
\[ e.g. 0.1 + a + 0.3 + b = 1 \]
\[ a + b = 0.6 \]  
A1 N2

(b) Evidence of correctly using \( E(X) = \sum xf(x) \)  
\[ e.g. 0 \times 0.1 + 1 \times a + 2 \times 0.3 + 3 \times b, 0.1 + a + 0.6 + 3b = 1.5 \]
Correct equation \( 0 + a + 0.6 + 3b = 1.5 \)  
\( a + 3b = 0.9 \)  
A1

Solving simultaneously gives  
\[ a = 0.45 \quad b = 0.15 \]  
A1A1 N3

7. (a) For using \( \sum p = 1 \)  
\[ p = 0.31 \]  
A1 N2

(b) For using \( E(X) = \sum xP(X=x) \)  
\[ E(X) = 1(0.4) + 2(0.31) + 3(0.2) + 4(0.07) + 5(0.02) \]
\[ = 2 \]  
A1 A2 N2

[7]

[6]
5.6 Discrete Probability Distributions

1. (a) evidence of using \( \sum p_i = 1 \) (M1)
correct substitution (A1)
e.g. \( 10k^2 + 3k + 0.6 = 1, \ 10k^2 + 3k - 0.4 = 0 \)
\( k = 0.1 \) (A2 N2)

(b) evidence of using \( E(X) = \sum p_i x_i \) (M1)
correct substitution (A1)
e.g. \(-1 \times 0.2 + 2 \times 0.4 + 3 \times 0.3\)
\( E(X) = 1.5 \) (A1 N2)

2. (a) Adding probabilities (M1)
Evidence of knowing that sum = 1 for probability distribution (R1)
e.g. Sum greater than 1, sum = 1.3, sum does not equal 1 (N2)

(b) Equating sum to 1 \( (3k + 0.7 = 1) \) (M1)
\( k = 0.1 \) (A1 N1)

(c) (i) \( P(X = 0) = \frac{0 + 1}{20} = \frac{1}{20} \) (M1)
(ii) evidence of using \( P(X > 0) = 1 - P(X = 0) \)
\( \left( \frac{4}{20} + \frac{5}{20} + \frac{10}{20} \right) \) (M1)
\( = \frac{19}{20} \) (A1 N2)

3. (a) three correct pairs (A1 A1 A1)
e.g. \( (2, 4), (3, 3), (4, 2), R2G4, R3G3, R4G2 \) (N3 3)

(b) \( p = \frac{1}{16}, \ q = \frac{2}{16}, \ r = \frac{2}{16} \) (A1 A1 A1)

(c) let \( X \) be the number of times the sum of the dice is 5
evidence of valid approach (M1)
e.g. \( X \sim B(n, p) \), tree diagram, 5 sets of outcomes produce a win
one correct parameter (A1)
e.g. \( n = 4, \ p = 0.25, q = 0.75 \)
Fred wins prize is \( P(X \geq 3) \) (A1)
appropriate approach to find probability (M1)
e.g. complement, summing probabilities, using a CDF function
correct substitution (A1)
e.g. \( 1 - 0.949\ldots, \ 1 - \frac{243}{256}, 0.046875 + 0.00390625, \frac{12}{256} + \frac{1}{256} \)
probability of winning = 0.0508 (13/256) (A1 N3 6)

[7]
[8]
[12]
4. (a) evidence of using mid-interval values (5, 15, 25, 35, 50, 67.5, 87.5) (M1)
   \( \sigma = 19.8 \text{ (cm)} \) A2 N3

(b) (i) \( Q_1 = 15, Q_3 = 40 \) (A1)(A1)
   \( IQR = 25 \text{ (accept any notation that suggests the interval 15 to 40)} \) A1 N3

(ii) **METHOD 1**
   - 60 % have a length less than \( k \) (A1)
   - \( 0.6 \times 200 = 120 \) (A1)
   - \( k = 30 \text{ (cm)} \) A1 N2

**METHOD 2**
   - \( 0.4 \times 200 = 80 \) (A1)
   - \( 200 - 80 = 120 \) (A1)
   - \( k = 30 \text{ (cm)} \) A1 N2

(c) \( l < 20 \text{ cm} \implies 70 \text{ fish} \) (M1)
   \( P(\text{small}) = \frac{70}{200} = 0.35 \) A1 N2

(d) \[
\begin{array}{|c|c|c|}
\hline
\text{Cost $X$} & 4 & 10 & 12 \\
\hline
P(X=x) & 0.35 & 0.565 & 0.085 \\
\hline
\end{array}
\]

(e) correct substitution (of their \( p \) values) into formula for \( E(X) \) (A1)
e.g. \( 4 \times 0.35 + 10 \times 0.565 + 12 \times 0.085 \)
\( E(X) = 8.07 \text{ (accept $8.07$)} \) A1 N2

[15]

5. (a) Using \( E(X) = \sum_{x=0}^{2} x P(X=x) \) (M1)

Substituting correctly \( E(X) = 0 \times \frac{3}{10} + 1 \times \frac{6}{10} + 2 \times \frac{1}{10} \)
\( = 0.8 \) A1 N2

(b) (i)

\[
\begin{array}{c}
\text{R}
\end{array}
\]

\[
\begin{array}{c}
\text{G}
\end{array}
\]

\[
\begin{array}{c}
\text{R}
\end{array}
\]

\[
\begin{array}{c}
\text{G}
\end{array}
\]

\[
\begin{array}{c}
\text{R}
\end{array}
\]

\[
\begin{array}{c}
\text{G}
\end{array}
\]

\[
\begin{array}{c}
\text{R}
\end{array}
\]

\[
\begin{array}{c}
\text{G}
\end{array}
\]

\[
\begin{array}{c}
\text{R}
\end{array}
\]

\[
\begin{array}{c}
\text{G}
\end{array}
\]

\[
\begin{array}{c}
\text{R}
\end{array}
\]

\[
\begin{array}{c}
\text{G}
\end{array}
\]

A1A1A1 N3
(ii) \[ P(Y = 0) = \frac{2}{5} \times \frac{1}{5} = \frac{2}{30} \] \[ A1 \]

\[ P(Y = 1) = P(RG) + P(GR) = \frac{4}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{4}{5} \] \[ M1 \]

\[ = \frac{16}{30} \] \[ A1 \]

\[ P(Y = 2) = \frac{4}{6} \times \frac{3}{5} = \frac{12}{30} \] \[ (A1) \]

For forming a distribution

<table>
<thead>
<tr>
<th>( y )</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(Y = y) )</td>
<td>( \frac{2}{30} )</td>
<td>( \frac{16}{30} )</td>
<td>( \frac{12}{30} )</td>
</tr>
</tbody>
</table>

\[ N4 \]

(c) \[ P(Bag A) = \frac{2}{6} \left( = \frac{1}{3} \right) \] \[ (A1) \]

\[ P(Bag B) = \frac{4}{6} \left( = \frac{2}{3} \right) \] \[ (A1) \]

For summing \( P(A \cap RR) \) and \( P(B \cap RR) \)

Substituting correctly \( P(RR) = \frac{1}{3} \times \frac{1}{10} + \frac{2}{3} \times \frac{12}{30} \)

\[ = 0.3 \] \[ A1 \] \[ N3 \]

(d) For recognising that \( P(1 \text{ or } 6 \mid RR) = P(A \mid RR) = \frac{P(A \cap RR)}{P(RR)} \)

\[ = \frac{1}{30} \div \frac{27}{90} \]

\[ = 0.111 \] \[ A1 \] \[ N2 \]

[19]

6. (a) (i) Attempt to set up sample space,

Any correct representation with 16 pairs \[ A2 \] \[ N3 \]

\[ eg \ 1,1 \ 2,1 \ 3,1 \ 4,1 \]

\[ 1,2 \ 2,2 \ 3,2 \ 4,2 \]

\[ 1,3 \ 2,3 \ 3,3 \ 4,3 \]

\[ 1,4 \ 2,4 \ 3,4 \ 4,4 \]

\[ 4 \quad x \quad x \quad x \quad x \]

\[ 3 \quad x \quad x \quad x \quad x \]

\[ 2 \quad x \quad x \quad x \quad x \]

\[ 1 \quad x \quad x \quad x \quad x \]

\[ 0 \quad 1 \quad 2 \quad 3 \quad 4 \]

\[ R \]

(ii) Probability of two 4s is \[ \frac{1}{16} \] \[ (= 0.0625) \] \[ A1 \] \[ N1 \]

44
(b)

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X = x)</td>
<td>$\frac{9}{16}$</td>
<td>$\frac{6}{16}$</td>
<td>$\frac{1}{16}$</td>
</tr>
</tbody>
</table>

(c) Evidence of selecting appropriate formula for $E(X)$ (M1)

\[ E(X) = \sum_0^2 x P(X = x), \quad E(X) = np \]

Correct substitution

\[ E(X) = 0 \times \frac{9}{16} + 1 \times \frac{6}{16} + 2 \times \frac{1}{16}, \quad E(X) = 2 \times \frac{1}{4} \]

\[ E(X) = \frac{8}{16} \left( \frac{1}{2} \right) \]

\[ A1 \quad N2 \quad [10] \]
5.7 Binomial Distribution

1. (a) (i) Attempt to find \( P(3H) = \left( \frac{1}{3} \right)^3 \) (M1)
   
   \[ \frac{1}{27} \]  
   
   A1  N2

   (ii) Attempt to find \( P(2H, 1T) \) (M1)
   
   \[ 3 \left( \frac{1}{3} \right)^2 \frac{2}{3} \]
   
   \[ \frac{2}{9} \]  
   
   A1  N2

(b) (i) Evidence of using \( np \) \( \left( \frac{1}{3} \times 12 \right) \) (M1)
   
   expected number of heads = 4  
   
   A1  N2

   (ii) 4 heads, so 8 tails (A1)
   
   E(winnings) = 4 \times 10 – 8 \times 6 (= 40 – 48) (M1)
   
   \[ = \$ 8 \]  
   
   A1  N1

2. (a) Evidence of binomial formula (M1)
   
   \[ P(X = 3) = \left( \binom{5}{3} \frac{1}{2}^3 \right) \]  
   
   \[ \frac{5}{16} (=0.313) \]  
   
   A1  N2

(b) **METHOD 1**

   P(at least one head) = 1 – P(X = 0) (M1)
   
   \[ = 1 - \left( \frac{1}{2} \right)^5 \]  
   
   A1

   \[ = \frac{31}{32} (=0.969) \]  
   
   A1  N2

   **METHOD 2**

   P(at least one head) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)
   
   \[ + P(X = 5) \]  
   
   (M1)

   \[ = 0.15625 + 0.3125 + 0.3125 + 0.15625 + 0.03125 \]  
   
   A1

   \[ = 0.969 \]  
   
   A1  N2

[10]
5.7 Binomial Distribution

1. (a) \( X \sim B(100, 0.02) \)
   
   \[ E(X) = 100 \times 0.02 = 2 \]
   
   A1 1

   (b) \( P(X = 3) = \binom{100}{3}(0.02)^3(0.98)^{97} \)

   \[ = 0.182 \]
   
   A1 2

   (c) **METHOD 1**
   
   \[ P(X > 1) = 1 - P(X \leq 1) = 1 - (P(X = 0) + P(X = 1)) \]

   \[ = 1 - ((0.98)^{100} + 100(0.02)(0.98)^{99}) \]

   \[ = 0.597 \]
   
   M1

   A1 2

   **METHOD 2**
   
   \[ P(X > 1) = 1 - P(X \leq 1) \]

   \[ = 1 - 0.40327 \]

   \[ = 0.597 \]
   
   A1 2

   \[ P(X \geq 1) \]

   \[ = 1 - P(X < 2) \]

   \[ = 1 - 0.67668 \]

   \[ = 0.323 \]
   
   M1 (ft)

2. (a) correct substitution into formula for \( E(X) \)

   e.g. \( 0.05 \times 240 \)

   \[ E(X) = 12 \]

   A1 N2 2

   (b) evidence of recognizing binomial probability (may be seen in part (a))

   e.g. \( \binom{240}{15}(0.05)^{15}(0.95)^{225}, X \sim B(240, 0.05) \)

   \[ P(X = 15) = 0.0733 \]

   A1 N2 2

   (c) \( P(X \leq 9) = 0.236 \)

   evidence of valid approach

   e.g. using complement, summing probabilities

   \[ P(X \geq 10) = 0.764 \]

   A1 N3 3

3. (a) evidence of recognizing binomial probability (may be seen in (b) or (c))

   e.g. probability = \( \binom{7}{4}(0.9)^4(0.1)^3 \), \( X \sim B(7, 0.9) \), complementary probabilities

   probability = 0.0230

   A1 N2

   (b) correct expression

   e.g. \( \binom{7}{4}p^4(1-p)^3 \)

   A1A1 N2

   (c) evidence of attempting to solve their equation

   e.g. \( \binom{7}{4}p^4(1-p)^3 = 0.15 \), sketch

   \[ p = 0.356, 0.770 \]

   A1A1 N3

[7]
4. (a) 36 outcomes (seen anywhere, even in denominator) (A1)
valid approach of listing ways to get sum of 5, showing at least two pairs (M1)
e.g. (1, 4)(2, 3), (1, 4)(4, 1), (1, 4)(4, 1), (2, 3)(3, 2), lattice diagram
\[ P(\text{prize}) = \frac{4}{36} \left( \frac{1}{9} \right) \] A1 N3

(b) recognizing binomial probability (M1)
e.g. \( B \left( \frac{8}{9}, \frac{1}{9} \right), \) binomial pdf,
\[ P(3 \text{ prizes}) = 0.0426 \] A1 N2

5. (a) (i) valid approach (M1)
e.g. \( np, 5 \times \frac{1}{5} \)
\[ E(X) = 1 \] A1 N2
(ii) evidence of appropriate approach involving binomial (M1)
e.g. \( X \sim B \left( \frac{5}{5}, \frac{1}{5} \right) \)
recognizing that Mark needs to answer 3 or more questions correctly (A1)
e.g. \( P(X \geq 3) \)
valid approach (M1)
e.g. \[ 1 - P(X \leq 2), P(X = 3) + P(X = 4) + P(X = 5) \]
P(\text{pass}) = 0.0579 A1 N3

(b) (i) evidence of summing probabilities to 1 (M1)
e.g. \( 0.67 + 0.05 + (a + 2b) + \ldots + 0.04 = 1 \)
some simplification that clearly leads to required answer (A1)
e.g. \( 0.76 + 4a + 2b = 1 \)
\[ 4a + 2b = 0.24 \] AG N0
(ii) correct substitution into the formula for expected value (A1)
e.g. \[ 0(0.67) + 1(0.05) + \ldots + 5(0.04) \]
some simplification (A1)
e.g. \[ 0.05 + 2a + 4b + \ldots + 5(0.04) = 1 \]
correct equation A1
e.g. \[ 13a + 5b = 0.75 \]
evidence of solving (M1)
\[ a = 0.05, b = 0.02 \] A1 A1 N4
(c) attempt to find probability Bill passes (M1)
e.g. \( P(Y \geq 3) \)
correct value 0.19 A1
Bill (is more likely to pass) A1 N0
6. (a) \[ E(X) = 2 \]
(b) evidence of appropriate approach involving binomial
\[ \text{e.g. } \binom{10}{3}(0.2)^3(0.8)^7, X \sim B(10, 0.2) \]
\[ P(X = 3) = 0.201 \]
(c) \textbf{METHOD 1}
\[ P(X \leq 3) = 0.10737 + 0.26844 + 0.30199 + 0.20133 (= 0.87912...) \]
evidence of using the complement (seen anywhere)
\[ 1 - \text{any probability}, P(X > 3) = 1 - P(X \leq 3) \]
\[ P(X > 3) = 0.121 \]
\textbf{METHOD 2}
recognizing that \( P(X > 3) = P(X \geq 4) \)
e.g. summing probabilities from \( X = 4 \) to \( X = 10 \)
correct expression or values
\[ \sum_{r=4}^{10} \binom{10}{r}(0.2)^{10-r}(0.8)^r \]
\[ 0.08808 + 0.02642 + 0.005505 + 0.000786 + 0.0000737 + 0.000004 + 0.0000001 \]
\[ P(X > 3) = 0.121 \]

7. (a) evidence of binomial distribution (may be seen in parts (b) or (c))
e.g. \( np, 100 \times 0.04 \)
mean = 4
(b) \[ P(X = 6) = \binom{100}{6}(0.04)^6(0.96)^{94} \]
\[ = 0.105 \]
(c) for evidence of appropriate approach
e.g. complement, \( 1 - P(X = 0) \)
\[ P(X = 0) = (0.96)^{100} = 0.01687... \]
\[ P(X \geq 1) = 0.983 \]

8. (a) evidence of using binomial probability
\[ \text{e.g. } P(X = 2) = \binom{7}{2}(0.18)^2(0.82)^5 \]
\[ P(X = 2) = 0.252 \]
(b) \textbf{METHOD 1}
evidence of using the complement
\[ \text{e.g. } 1 - P(X = 3) \]
\[ P(X \leq 1) = 0.632 \]
\[ P(X \geq 2) = 0.368 \]
\textbf{METHOD 2}
evidence of attempting to sum probabilities
\[ \text{e.g. } P(2 \text{ heads}) + P(3 \text{ heads}) + \ldots + P(7 \text{ heads}) \]
correct values for each probability
\[ \text{e.g. } 0.252 + 0.0923 + 0.0203 + 0.00267 + 0.0002 + 0.0000061 \]
\[ P(X \geq 2) = 0.368 \]
9. (a) \[ X \sim B(100, 0.02) \]
   \[ E(X) = 100 \times 0.02 = 2 \]  
   A1 N1

(b) \[ P(X = 3) = \binom{100}{3} (0.02)^3 (0.98)^{97} \]  
   \[ = 0.182 \]  
   A1 N2

(c) **METHOD 1**

\[ P(X > 1) = 1 - P(X \leq 1) = 1 - (P(X = 0) + P(X = 1)) \]

\[ = 1 - ((0.98)^{100} + 100(0.02)(0.98)^{99}) \]

\[ = 0.597 \]  
   M1 N2

**METHOD 2**

\[ P(X > 1) = 1 - P(X \leq 1) \]

\[ = 1 - 0.40327 \]  
   A1 N2

\[ P(X \geq 1) \]

\[ = 1 - P(X \leq 2) = 1 - 0.67668 \]

\[ = 0.323 \]  
   M1(FT) N0

10. (a)  

\[ \begin{align*}
\text{First die in pair} & \quad \text{Second die in pair} \\
\text{four} & \quad \frac{1}{6} \\
\frac{1}{6} & \quad \text{four} \\
\frac{5}{6} & \quad \text{not four} \\
\frac{5}{6} & \quad \text{not four} \\
\frac{1}{6} & \quad \text{four} \\
\frac{5}{6} & \quad \text{not four} \\
\frac{5}{6} & \quad \text{not four}
\end{align*} \]

   A1A1A1 N3

(b) \[ P(E) = \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} \left( = \frac{5}{36} + \frac{5}{36} \right) \]

\[ = \frac{10}{36} \left( = \frac{5}{18} \text{ or } 0.278 \right) \]  
   A1 N3

(c) **Evidence of recognizing the binomial distribution**

\[ \text{eg } X \sim B \left( 5, \frac{5}{18} \right) \text{ or } p = \frac{5}{18}, q = \frac{13}{18} \]

\[ P(X = 3) = \binom{5}{3} \left( \frac{5}{18} \right)^3 \left( \frac{13}{18} \right)^2 \]  
   (or other evidence of correct setup)  
   A1 N3
(d) **METHOD 1**

Evidence of using the complement  
\[ \text{eg } P(X \geq 3) = 1 - P(X \leq 2) \]
Correct value \[ 1 - 0.865 \]  
\[ = 0.135 \]  
(A1)  
A1 N2

**METHOD 2**

Evidence of adding correct probabilities  
\[ \text{eg } P(X \geq 3) = P(X=3) + P(X=4) + P(X=5) \]
Correct values \[ 0.1118 + 0.02150 + 0.001654 \]  
\[ = 0.135 \]  
A1 N2


11. \( p(\text{Red}) = \frac{35}{40} = \frac{7}{8} \)
\( p(\text{Black}) = \frac{5}{40} = \frac{1}{8} \)

(a) (i) \( p(\text{one black}) = \left( \frac{8}{1} \right) \left( \frac{1}{8} \right) \left( \frac{7}{8} \right)^7 \)  
(M1)(A1)
\[ = 0.393 \] to 3 sf  
(A1) 3

(ii) \( p(\text{at least one black}) = 1 - p(\text{none}) \)  
(M1)
\[ = 1 - \left( \frac{8}{0} \right) \left( \frac{1}{8} \right)^0 \left( \frac{7}{8} \right)^8 \]  
(A1)
\[ = 1 - 0.344 \]
\[ = 0.656 \]  
(A1) 3

(b) 400 draws: expected number of blacks \[ \frac{400}{8} \]  
\[ = 50 \]  
(A1) 2

12. (a) \( p(4 \text{ heads}) = \left( \frac{8}{4} \right)^4 \left( \frac{1}{2} \right)^{8-4} \)  
(M1)
\[ = \frac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4} \times \left( \frac{1}{2} \right)^8 \]
\[ = \frac{70}{256} \approx 0.273 \) (3 sf)  
(A1) 2

(b) \( p(3 \text{ heads}) = \left( \frac{8}{3} \right)^3 \left( \frac{1}{2} \right)^{8-3} = \frac{8 \times 7 \times 6}{1 \times 2 \times 3} \times \left( \frac{1}{2} \right)^8 \)
\[ = \frac{56}{256} \approx 0.219 \) (3 sf)  
(A1) 1

(c) \( p(5 \text{ heads}) = p(3 \text{ heads}) \) (by symmetry)  
(M1)
\( p(3 \text{ or 4 or 5 heads}) = p(4) + 2p(3) \)  
(M1)
\[ = \frac{70 + 2 \times 56}{256} = \frac{182}{256} \]
\[ \approx 0.711 \) (3 sf)  
(A1) 3
5.8 Normal Distribution

1. (a) 

![Normal Distribution Graph]

*Note:* Award A1 for vertical line to right of mean, A1 for shading to right of their vertical line.

(b) evidence of recognizing symmetry (M1)
e.g. 105 is one standard deviation above the mean so $d$ is one standard deviation below the mean, shading the corresponding part,

$$105 - 100 = 100 - d$$

$d = 95$ 

(c) evidence of using complement (M1)
e.g. $1 - 0.32, 1 - p$

$$P(d < X < 105) = 0.68$$

2. **METHOD 1**

(a) $\sigma = 10$

$$1.12 \times 10 = 11.2$$

$$11.2 + 100$$

$$x = 111.2$$

(b) $100 - 11.2$

$$= 88.8$$

3. **METHOD 2**

(a) $\sigma = 10$

Evidence of using standardisation formula (M1)

$$\frac{x - 100}{10} = 1.12$$

$$x = 111.2$$

(b) $\frac{100 - x}{10} = 1.12$

$$x = 88.8$$

3. (a) Evidence of using the complement e.g. $1 - 0.06$

$p = 0.94$ 

(b) For evidence of using symmetry (M1)

Distance from the mean is 7

e.g. diagram, $D = \text{mean} - 7$

$$D = 10$$

(c) $P(17 < H < 24) = 0.5 - 0.06$

$$= 0.44$$

$$E(\text{trees}) = 200 \times 0.44$$

$$= 88$$
5.8 Normal Distribution

1. (a) 0.0668
(b) Using the standardized value 1.645
   \( k = 26.1 \text{ kg} \)
(c)

2. (a) \( P(H < 153) = 0.705 \Rightarrow z = 0.538(836...) \) (A1)
   Standardizing \( \frac{153-\mu}{5} \) (A1)
   Setting up their equation \( 0.5388... = \frac{153-\mu}{5} \) M1
   \( \mu = 150.30... \)
   = 150 (to 3sf) A1 N3
(b) \( Z = \frac{153-\mu}{5} = 1.138... \)  (accept 1.14 from \( \mu = 150.3 \), or 1.2 from \( \mu = 150 \)) (A1)
   \( P(Z > 1.138) = 0.128 \)  (accept 0.127 from \( z = 1.14 \), or 0.115 from \( z = 1.2 \)) A1 N2

3. (a) \( z = \frac{180 - 160}{20} = 1 \) (A1)
   \( \phi(1) = 0.8413 \) (A1)
   \( P(\text{height} > 180) = 1 - 0.8413 \)
   = 0.159 A1 N3
(b) \( z = -1.1800 \) (A1)
   Setting up equation \(-1.18 = \frac{d - 160}{20}\) (M1)
   \( d = 136 \) A1 N3
4. \( X \sim N(\mu, \sigma^2) \), \( P(X < 3) = 0.2 \), \( P(X > 8) = 0.1 \)

\( P(X < 8) = 0.9 \) \hspace{1cm} (M1)

Attempt to set up equations \hspace{1cm} (M1)

\[
\frac{3 - \mu}{\sigma} = -0.8416, \quad \frac{8 - \mu}{\sigma} = 1.282
\]

\[
3 - \mu = -0.8416\sigma
\]

\[
8 - \mu = 1.282\sigma
\]

\[
5 = 2.1236\sigma
\]

\[
\sigma = 2.35, \quad \mu = 4.99
\]

5. \( X \sim N(\mu, \sigma^2) \), \( P(X > 90) = 0.15 \) and \( P(X < 40) = 0.12 \) \hspace{1cm} (M1)

Finding standardized values \( 1.036, -1.175 \) \hspace{1cm} A1A1

Setting up the equations \( 1.036 = \frac{90 - \mu}{\sigma}, -1.175 = \frac{40 - \mu}{\sigma} \) \hspace{1cm} (M1)

\[
\mu = 66.6, \quad \sigma = 22.6
\]

6. (a) \( \sigma = 3 \) \hspace{1cm} (A1)

Evidence of attempt to find \( P(X \leq 24.5) \) \hspace{1cm} (M1)

\[ e.g. \ z = 1.5, \ \frac{24.5 - 20}{3} \]

\( P(X \leq 24.5) = 0.933 \)

\[ A1 \quad N3 \quad 3 \]

(b) (i)

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{diagram.png}
\end{figure}

\[ A1A1 \quad N2 \]

(ii) \( z = 1.03(64338) \) \hspace{1cm} (A1)

Attempt to set up an equation \hspace{1cm} (M1)

\[ e.g. \ \frac{k - 20}{3} = 1.0364, \ \frac{k - 20}{3} = 0.85 \]

\[ k = 23.1 \]

\[ A1 \quad N3 \quad 5 \]
7. (a) symmetry of normal curve
   e.g. \( P(X < 25) = 0.5 \)
   \( P(X > 27) = 0.2 \)
(b) **METHOD 1**
   finding standardized value
   e.g. \( \frac{27 - 25}{\sigma} \)
   evidence of complement
   e.g. \( 1 - p \), \( P(X < 27), 0.8 \)
   finding \( z \)-score
   e.g. \( z = 0.84 \ldots \)
   attempt to set up equation involving the standardized value
   e.g. \( 0.84 = \frac{27 - 25}{\sigma}, 0.84 = \frac{X - \mu}{\sigma} \)
   \( \sigma = 2.38 \)
**METHOD 2**
   set up using normal CDF function and probability
   e.g. \( P(25 < X < 27) = 0.3, P(X < 27) = 0.8 \)
   correct equation
   e.g. \( P(25 < X < 27) = 0.3, P(X > 27) = 0.2 \)
   attempt to solve the equation using GDC
   e.g. solver, graph, trial and error (more than two trials must be shown)
   \( \sigma = 2.38 \)

8. (a) evidence of attempt to find \( P(X \leq 475) \)
    e.g. \( P(Z \leq 1.25) \)
    \( P(X \leq 475) = 0.894 \)
(b) evidence of using the complement
    e.g. \( 0.73, 1 - p \)
    \( z = 0.6128 \)
    setting up equation
    e.g. \( \frac{a - 450}{20} = 0.6128 \)
    \( a = 462 \)
9. (a) evidence of appropriate approach
   \( e.g. \ 1 - 0.85, \) diagram showing values in a normal curve
   \[ P(w \geq 82) = 0.15 \]

(b) (i) \( z = -1.64 \)

(ii) evidence of appropriate approach
   \( e.g. \ -1.64 = \frac{x - \mu}{\sigma}, \frac{68 - 76.6}{\sigma} \)
   correct substitution
   \( e.g. \ -1.64 = \frac{68 - 76.6}{\sigma} \)
   \( \sigma = 5.23 \)

(c) (i) \( 68.8 \leq \text{weight} \leq 84.4 \)

(ii) evidence of appropriate approach
   \( e.g. \ P(-1.5 \leq z \leq 1.5), \ P(68.76 < y < 84.44) \)
   \( P(\text{qualify}) = 0.866 \)

(d) recognizing conditional probability
   \( e.g. \ P(A|B) = \frac{P(A \cap B)}{P(B)} \)
   \( P(\text{woman and qualify}) = 0.25 \times 0.7 \)
   \( P(\text{woman}|\text{qualify}) = \frac{0.25 \times 0.7}{0.866} \)
   \( P(\text{woman}|\text{qualify}) = 0.202 \)

10. \( A \sim N(46, 10^2) \) \( B \sim N(\mu, 12^2) \)

(a) \( P(A > 60) = 0.0808 \)

(b) correct approach
   \( e.g. \ P \left( Z < \frac{60 - \mu}{12} \right) = 0.85, \) sketch
   \( \frac{60 - \mu}{12} = 1.036... \)
   \( \mu = 47.6 \)

(c) (i) route A

(ii) METHOD 1
   \( P(A < 60) = 1 - 0.0808 = 0.9192 \)
   valid reason
   \( e.g. \) probability of \( A \) getting there on time is greater than probability of \( B \)
   \( 0.9192 > 0.85 \)

METHOD 2
   \( P(B > 60) = 1 - 0.85 = 0.15 \)
   valid reason
   \( e.g. \) probability of \( A \) getting there late is less than probability of \( B \)
   \( 0.0808 < 0.15 \)
(d) (i) let $X$ be the number of days when the van arrives before 07:00

\[ P(X = 5) = (0.85)^5 \]
\[ = 0.444 \] (A1) 

\[ A1 \quad N2 \]

(ii) **METHOD 1**

evidence of adding correct probabilities (M1)

\[ e.g. \ P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5) \]

\[ \text{correct values} \ 0.1382 + 0.3915 + 0.4437 \] (A1)

\[ P(X \geq 3) = 0.973 \] (A1) \quad N3

**METHOD 2**

evidence of using the complement (M1)

\[ e.g. \ P(X \geq 3) = 1 - P(X \leq 2), \ 1 - p \]

\[ \text{correct values} \ 1 - 0.02661 \] (A1)

\[ P(X \geq 3) = 0.973 \]

\[ A1 \quad N3 \]

11. $X \sim N(7, 0.5^2)$

(a) (i) 

\[ z = 2 \] (M1)

\[ P(X < 8) = P(Z < 2) = 0.977 \] (A1) \quad N2

(ii) evidence of appropriate approach (M1)

\[ e.g. \ \text{symmetry}, \ z = -2 \]

\[ P(6 < X < 8) = 0.954 \] (tables 0.955) (A1) \quad N2

(b) (i) 

\[ z = -1.645 \] (A1)

\[ \frac{d - 7}{0.5} = -1.645 \] (M1)

\[ d = 6.18 \]

\[ A1 \quad A1 \quad N2 \]

(ii) 

\[ z = -1.645 \] (A1)

\[ \frac{d - 7}{0.5} = -1.645 \] (M1)

\[ d = 6.18 \]

\[ A1 \quad N3 \]

(c) $Y \sim N(\mu, 0.5^2)$

\[ P(Y < 5) = 0.2 \] (M1)

\[ z = -0.84162... \]

\[ \frac{5 - \mu}{0.5} = -0.8416 \] (M1)

\[ \mu = 5.42 \]

\[ A1 \quad N3 \]

[13]
12. (a) 

(b) **METHOD 1**

\[ P(X < r) = 0.1292 \]  
\[ r = 6.56 \]

\[ 1 - 0.1038 (= 0.8962) \text{ (may be seen later)} \]

\[ P(X < t) = 0.8962 \]  
\[ t = 7.16 \]

**METHOD 2**

finding \( z \)-values \(-1.130..., 1.260...\)

evidence of setting up one standardized equation

\[ e.g. \quad \frac{r - 6.84}{0.25} = -1.13..., \quad t = 1.260 \times 0.25 + 6.84 \]

\[ r = 6.56, \quad t = 7.16 \]

13. (a) evidence of approach

\[ e.g. \text{ finding } 0.84..., \text{ using } \frac{23.7 - 21}{\sigma} \]

correct working

\[ e.g. \quad 0.84... = \frac{23.7 - 21}{\sigma}, \text{ graph} \]

\[ \sigma = 3.21 \]

(b) (i) evidence of attempting to find \( P(X < 25.4) \)

\[ e.g. \quad \text{using } z = 1.37 \]

\[ P(X < 25.4) = 0.915 \]

(ii) evidence of recognizing symmetry

\[ e.g. \quad b = 21 - 4.4, \text{ using } z = -1.37 \]

\[ b = 16.6 \]
Note: Candidates may be using tables in this question, which leads to a variety of values. Accept reasonable answers that are consistent with working shown.

\[ W \sim N(2.5, 0.3^2) \]

(a) (i) \[ z = -1.67 \] (accept 1.67) (A1)

\[ P(W < 2) = 0.0478 \] (accept answers between 0.0475 and 0.0485) A1 N2

(ii) \[ z = 1 \] (A1)

\[ P(W > 2.8) = 0.159 \] A1 N2

(iii)

\[ \text{A1A1 N2} \]

(iv) Evidence of appropriate calculation M1

eg \[ 1 - (0.047790 + 0.15866), 0.8413 - 0.0478 \]

\[ P = 0.7936 \] AG N0

(b) (i) \[ X \sim B(10, 0.7935...) \]

Evidence of calculation M1

eg \[ P(X = 10) = (0.7935...)^{10} \]

\[ P(X = 10) = 0.0990 \] (3 sf) A1 N1

(ii) \textbf{METHOD 1}

Recognizing \[ X \sim B(10, 0.7935...) \] (may be seen in (i)) (M1)

\[ P(X \leq 6) = 0.1325... \text{ (or } P(X = 1) + ... + P(X = 6)) \] (A1)

evidence of using the complement (M1)

eg \[ P(X \geq 7) = 1 - P(X \leq 6), P(X \geq 7) = 1 - P(X < 7) \]

\[ P(X \geq 7) = 0.867 \] A1 N3

\textbf{METHOD 2}

Recognizing \[ X \sim B(10, 0.7935...) \] (may be seen in (i)) (M1)

For adding terms from \( P(X = 7) \) to \( P(X = 10) \) (M1)

\[ P(X \geq 7) = 0.209235 + 0.301604 + 0.257629 + 0.099030 \]

\[ = 0.867 \] A1 N3

[13]
15. **Notes:** Accept any suitable notation, as long as the candidate’s intentions are clear. The following symbols will be used in the markscheme.

Girls’ height $G \sim N(155, 10^2)$, boys’ height $B \sim N(160, 12^2)$

**Height $H$, Female $F$, Male $M$.**

(a) $P(G > 170) = 1 - P(G < 170)$ (A1)

$$P(G > 170) = P\left(Z < \frac{170 - 155}{10}\right)$$ (A1)

$$P(G > 170) = 1 - \Phi (1.5) = 1 - 0.9332 = 0.0668$$ (A1 N3)

(b) $z = -1.2816$ (A1)

Correct calculation (eg $x = 155 + (-1.282 \times 10)$) (A1)

$x = 142$ (A1 N3)

(c) Calculating one variable (A1)

eg $P(B < r) = 0.95$, $z = 1.6449$

$r = 160 + 1.645(12) = 179.74$

$= 180$ (A1 N2)

Any valid calculation for the second variable, including use of symmetry (A1)

eg $P(B < q) = 0.05$, $z = -1.6449$

$q = 160 - 1.645(12) = 140.26$

$= 140$ (A1 N2)

(d) $P(M \cap (B > 170)) = 0.4 \times 0.2020$, $P(F \cap (G > 170)) = 0.6 \times 0.0668$ (A1)(A1)

$$P(H > 170) = 0.0808 + 0.04008$$ (A1)

$= 0.12088 = 0.121$ (3 sf) (A1 N2)

(e) $P(F \mid H > 170) = \frac{P(F \cap (H > 170))}{P(H > 170)}$ (M1)

$$= \frac{0.60 \times 0.0668}{0.121} = \frac{0.0401}{0.121} \text{ or } \frac{0.04008}{0.1208}$$ (A1)

$= 0.332$ (A1 N1)
16. (i) \( P(X > 3200) = P(Z > 0.4) \)  
\[ = 1 - 0.6554 = 0.345 \] (M1) (A1) (N2)

(ii) \( P(2300 < X < 3300) = P(-1.4 < Z < 0.6) \)  
\[ = 0.4192 + 0.2257 \] (M1)  
\[ = 0.645 \] (A1)

\( P(\text{both}) = (0.645)^2 = 0.416 \) (A1) (N2)

(iii) \( 0.7422 = P(Z < 0.65) \)  
\[ \frac{d - 3000}{500} = 0.65 \] (A1)  
\( d = \$3325 \) (Accept $3325.07) (A1) (N3) [8]

17. (a) \[ z = \frac{185 - 170}{20} = 0.75 \] (M1)(A1)

\( P(Z < 0.75) = 0.773 \) (A1) (N3)

(b) \( z = -0.47 \) (may be implied) (A1)

\[ -0.47 = \frac{d - 170}{20} \] (M1)  
\( d = 161 \) (A1) (N3) [6]

18. (a) (i) \( a = -1 \) (A1)
\( b = 0.5 \) (A1)

(ii) (a) \( 0.841 \) (A2)

(b) \[ 0.6915 - 0.1587 = 0.533 \] (3 sf) (M1)
\[ = 0.533 \] (N2) 6 (A1)

(b) (i) Sketch of normal curve (A1)(A1)

(ii) \( c = 0.647 \) (A2) 4 [10]
19. \(X \sim N(80, 8^2)\)

(a) \(P(X < 72) = P(Z < -1)\)  
\[= 1 - 0.8413\]  
\[= 0.159\]  
**OR**  
\(P(X < 72) = 0.159\)  
**(G2)**

(b) (i) \(P(72 < X < 90) = P(-1 < Z < 1.25)\)  
\[= 0.3413 + 0.3944\]  
\[= 0.736\]  
**OR**  
\(P(72 < X < 90) = 0.736\)  
**(G3)**

(ii)

(c) 4% fail in less than \(x\) months  
\[\Rightarrow x = 80 - 8 \times \Phi^{-1}(0.96)\]  
\[= 80 - 8 \times 1.751\]  
\[= 66.0\] months  
**OR**  
\(x = 66.0\) months  
**(G3)**

20. (a) \(P(M \geq 350) = 1 - P(M < 350) = 1 - P\left(Z < \frac{350 - 310}{30}\right)\)  
\[= 1 - P(Z < 1.333) = 1 - 0.9088\]  
\[= 0.0912\] (accept 0.0910 to 0.0920)  
**OR**  
\(P(M \geq 350) = 0.0912\)  
**(G2)**

(b) \[P(Z < 1.96) = 0.025\]  
\[1.96 \times (30) = 58.8\]  
\[310 - 58.8 < M < 310 + 58.8 \Rightarrow a = 251, b = 369\]  
**OR**  
\(251 < M < 369\)  
**(G3)**
21. \[\text{Note: Where accuracy is not specified, accept answers with greater than 3 sf accuracy, provided they are correct as far as 3 sf}\]

(a) \[z = \frac{197 - 187.5}{9.5} = 1.00\] (M1)

\[P(Z > 1) = 1 - \Phi(1) = 1 - 0.8413 = 0.1587\]
\[= 0.159 \text{ (3 sf)}\] (A1)
\[= 15.9\%\] (A1)

OR

\[P(H > 197) = 0.159\] (G2)
\[= 15.9\%\] (A1) 3

(b) Finding the 99th percentile

\[\Phi(a) = 0.99 \Rightarrow a = 2.327 \text{ (accept 2.33)}\] (A1)
\[\Rightarrow 99\% \text{ of heights under } 187.5 + 2.327(9.5) = 209.6065\]
\[= 210 \text{ (3 sf)}\] (A1)

OR

99\% of heights under 209.6 = 210 cm (3 sf) (G3)

Height of standard doorway = 210 + 17 = 227 cm (A1) 4

22. (a) (These answers may be obtained from a calculator or by finding \(z\) in each case and the corresponding area.)

\(M \sim N(750, 625)\)

(i) \[P(M < 740 \text{ g}) = 0.345\] (G2)

\[z = -0.4 \quad P(z < -0.4) = 0.345\] (A1)(A1)

(ii) \[P(M > 780 \text{ g}) = 0.115\] (G2)

\[z = 1.2 \quad P(z > 1.2) = 1 - 0.885 = 0.115\] (A1)(A1)

(iii) \[P(740 < M < 780) = 0.540\] (G1)

\[1 - (0.345 + 0.115) = 0.540\] (A1) 5

(b) Independent events

Therefore, \(P(\text{both} < 740) = 0.345^2\) (M1)
\[= 0.119\] (A1) 2

(c) 70\% have mass < 763 g

Therefore, 70\% have mass of at least 750 – 13
\[x = 737 \text{ g}\] (A1) 2
23. (a) Let \( X \) be the random variable for the IQ.

\[ X \sim N(100, 225) \]

\[ P(90 < X < 125) = P(-0.67 < Z < 1.67) \quad \text{(M1)} \]

\[ = 0.701 \]

70.1 percent of the population (accept 70 percent). \( \text{(A1)} \)

OR

\[ P(90 < X < 125) = 70.1\% \quad \text{(G2)} \]

(b) \( P(X \geq 125) = 0.0475 \) (or 0.0478) \( \text{(M1)} \)

\[ P(\text{both persons having IQ} \geq 125) = (0.0475)^2 \text{ (or } (0.0478)^2) \quad \text{(M1)} \]

\[ = 0.00226 \text{ (or } 0.00228) \quad \text{(A1)} \]

(c) Null hypothesis \( (H_0) \): mean IQ of people with disorder is 100

Alternative hypothesis \( (H_1) \): mean IQ of people with disorder is less than 100

\[ P(\bar{X} < 95.2) = P\left( Z < \left( \frac{95.2 - 100}{15} \right) \right) = P(Z < -1.6) = 1 - 0.9452 \]

\[ = 0.0548 \quad \text{(A1)} \]

The probability that the sample mean is 95.2 and the null hypothesis true is 0.0548 > 0.05. Hence the evidence is not sufficient. \( \text{(R1)} \)

24. (a) \( Z = \frac{25 - 25.7}{0.50} = -1.4 \quad \text{(M1)} \)

\[ P(Z < -1.4) = 1 - P(Z < 1.4) \]

\[ = 1 - 0.9192 \]

\[ = 0.0808 \quad \text{(A1)} \]

OR

\[ P(W < 25) = 0.0808 \quad \text{(G2)} \]

(b) \( P(Z < -a) = 0.025 \Rightarrow P(Z < a) = 0.975 \Rightarrow a = 1.960 \quad \text{(A1)} \)

\[ \frac{25 - \mu}{0.50} = -1.96 \Rightarrow \mu = 25 + 1.96 (0.50) \quad \text{(M1)} \]

\[ = 25 + 0.98 = 25.98 \quad \text{(A1)} \]

\[ = 26.0 \text{ (3 sf)} \quad \text{(AG)} \]

OR

\[ \frac{25.0 - 26.0}{0.50} = -2.00 \quad \text{(M1)} \]

\[ P(Z < -2.00) = 1 - P(Z < 2.00) \]

\[ = 1 - 0.9772 = 0.0228 \quad \text{(A1)} \]

\[ \approx 0.025 \quad \text{(A1)} \]

OR

\[ \mu = 25.98 \quad \text{(G2)} \]

\[ \Rightarrow \text{mean} = 26.0 \text{ (3 sf)} \quad \text{(A1)(AG)} \]

(c) Clearly, by symmetry \( \mu = 25.5 \quad \text{(A1)} \)

\[ Z = \frac{25.0 - 25.5}{\sigma} = -1.96 \Rightarrow 0.5 = 1.96\sigma \quad \text{(M1)} \]

\[ \Rightarrow \sigma = 0.255 \text{ kg} \quad \text{(A1)} \]
(d) On average, \( \frac{\text{cement saving}}{\text{bag}} = 0.5 \text{ kg} \) \( \text{(A1)} \)

\[ \frac{\text{cost saving}}{\text{bag}} = 0.5(0.80) = 0.40 \] \( \text{(M1)} \)

To save $5000 takes \( \frac{5000}{0.40} = 12500 \text{ bags} \) \( \text{(A1)} \) \(3\)

25. (a) Let \( X \) be the lifespan in hours

\( X \sim N(57, 4.4^2) \)

\[ a = -0.455 \text{ (3 sf)} \] \( \text{(A1)} \)
\[ b = 0.682 \text{ (3 sf)} \] \( \text{(A1)} \)

(i) \( a = -0.455 \) \( \text{(3 sf)} \) \( \text{(A1)} \)
\[ b = 0.682 \text{ (3 sf)} \] \( \text{(A1)} \)

(ii) (a) \( P(X > 55) = P(Z > -0.455) \)
\[ = 0.675 \text{ (A1)} \]

(b) \( P(55 \leq X \leq 60) = P \left( \frac{2}{4.4} \leq Z \leq \frac{3}{4.4} \right) \)
\[ \approx P(0.455 \leq Z \leq 0.682) \]
\[ \approx 0.6754 + 0.752 - 1 \]
\[ = 0.428 \text{ (3sf)} \] \( \text{(A1)} \)

\text{OR}

\[ P(55 \leq X \leq 60) = 0.428 \text{ (3 sf)} \] \( \text{(G2)} \) \(5\)

(b) 90\% have died \( \Rightarrow \) shaded area = 0.9

\[ t = 57 + (4.4 \times 1.282) \] \( \text{(M1)} \)
\[ = 57 + 5.64 \] \( \text{(A1)} \)
\[ = 62.6 \text{ hours} \] \( \text{(A1)} \)

\text{OR}

\[ t = 62.6 \text{ hours} \] \( \text{(G3)} \) \(5\)
26. (a) \( P(\text{speed} > 50) = 0.3 = 1 - \Phi\left(\frac{50 - \mu}{10}\right) \) (A1)

Hence, \( \frac{50 - \mu}{10} = \Phi^{-1}(0.7) \) (M1)

\[ \mu = 50 - 10\Phi^{-1}(0.7) \]
\[ = 44.75599 \ldots = 44.8 \text{ km/h (3 sf) (accept 44.7)} \] (AG) 3

(b) \( H_1: \) “the mean speed has been reduced by the campaign”. (A1) 1

c) One-tailed; because \( H_1 \) involves only “<”. (A2) 2

d) For a one-tailed test at 5% level, critical region is \( Z < \mu_m - 1.64\sigma_m \) (accept \(-1.65\sigma_m\)) (M1)

Now, \( \mu_m = \mu = 44.75\ldots; \sigma_m = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{25}} = 2 \) (\( allow \ ft \)) (A1)

So test statistic is \( 44.75\ldots -1.64 \times 2 = 41.47 \) (A1)

Now 41.3 < 41.47 so reject \( H_0 \), yes. (A1) 4

27. (a) Area \( A = 0.1 \) (A1) 1

(b) Since \( p(X \geq 12) = p(X \leq 8) \), then 8 and 12 are symmetrically disposed around the mean.

Thus mean = \( \frac{8 + 12}{2} \) = 10 (M1) (A1) 5

(c) \( \Phi\left(\frac{12-10}{\sigma}\right) = 0.9 \) (A1)(M1)(A1)

\[ \Rightarrow \frac{2}{\sigma} = 1.282 \text{ (or 1.28)} \] (A1)

\[ \sigma = \frac{2}{1.282} \text{ (or } \frac{2}{1.28} \text{)} \] (A1)

\[ = 1.56 \text{ (3 sf)} \] (AG) 5

(d) \( p(X \leq 11) = p\left(Z \leq \frac{11 - 10}{1.561}\right) \) (or 1.56) (M1)(A1)

\[ = p(Z \leq 0.6407) \text{ (or 0.641 or 0.64)} \] (A1)

\[ = \Phi(0.6407) \] (M1)

\[ = 0.739 \text{ (3 sf)} \] (A1) 5