1. Do not use a calculator to answer this question.
   (a) Find the exact value of $3 \log(5) - \log(20) + \log(16)$.
   (b) Given that $x = \ln 2$, $y = \ln 3$ and $z = \ln 5$, express $\ln \left( \frac{45}{4} \right)$ in terms of $x$, $y$ and $z$.
   (c) If $\ln K = 2 - \ln c$, find and simplify an expression for $K$ in terms of $c$.
      (accessible to students on the path to grade 3 or 4) [6 marks]

2. Do not use a calculator to answer this question.
   Solve the following equation:
   $\log_2 (x + 2) - \log_2 x = 3$
      (accessible to students on the path to grade 3 or 4) [3 marks]

3. Find the exact solutions of the equation,
   $3e^{2x} - 7e^x + 2 = 0$
      (accessible to students on the path to grade 5 or 6) [5 marks]

4. The diagram shows the graph with equation $y = C + Ae^{-kt}$. The graph passes through the point $P(2, 3)$.
   
   ![Diagram of graph with points labeled P(2, 3) and y = 2]
(a) Write down the value of $C$ and the value of $A$.

(b) Find the exact value of $k$.

\( \text{(accessible to students on the path to grade 5 or 6) [5 marks]} \)

5. (a) The population of bacteria increases according to an exponential model, \( N = A \times b^{kt} \), where $N$ is the number of bacteria after $t$ minutes and $A$ and $b$ are positive constants. Given that initially there were 50 bacteria and that after three minutes the number has grown to 270,

(i) Write down the value of $A$.

(ii) Show that, to three significant figures, $b^k = 1.75$.

(iii) Find the size of the population after five minutes.

(b) After five minutes the population growth slows down, so that now it follows the new model,

\[ N = 2000 - Me^{-0.47t}. \]

(i) Find the value of $M$.

(ii) According to this model, the size of the population approaches a limit in the long term. Find this limit.

(iii) How long does it take for the population size to reach 1999? Give your answer to the nearest minute.

\( \text{(accessible to students on the path to grade 5 or 6) [11 marks]} \)