Self-assessment: 18 Probability distributions

1. The random variable $X$ has distribution shown in the table below:

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X=x)$</td>
<td>$p$</td>
<td>$\frac{1}{5}$</td>
<td>$\frac{1}{10}$</td>
<td>$2p$</td>
<td>$\frac{2}{5}$</td>
</tr>
</tbody>
</table>

(a) Find the value of $p$.

(b) Find $P(X \geq 4)$.

(c) Find the expected value of $X$.

(accessible to students on the path to grade 3 or 4) [8 marks]

2. The amount of coffee dispensed by a machine follows normal distribution with mean 150 ml and standard deviation 5 ml.

(a) Calculate the probability that the machine dispenses less than 142 ml of coffee.

(b) Find the value of $a$ if 20% of cups contain more than $a$ ml of coffee.

(accessible to students on the path to grade 3 or 4) [5 marks]

3. A die is biased so that the probability of rolling a six is 0.12. The die is rolled seven times.

(a) Find the probability that no sixes are rolled.

(b) Find the expected number of sixes.

(c) Find the variance of the number of sixes.

(d) Find the probability that more than four sixes are rolled.

(accessible to students on the path to grade 5 or 6) [9 marks]

4. Random variable $X$ follows binomial distribution $B(6, p)$ and $P(X = 4) = 0.261$.

(a) Find the possible values of $p$.

(b) For the maximum value from (a), find $P(X \leq 2)$.

(c) For the maximum value from (a), find $E(X)$ and $\text{Var}(X)$.

(accessible to students on the path to grade 7) [8 marks]