3.1 - CIRCLE BASICS

RADIANS ⇒ Alternative way of measuring degrees.
⇒ \( \pi \) radians = 180° [2\( \pi \) = 360°, \( \pi \) 2 = 90°]

OTHER RULES ⇒

SECTOR:

ARC LENGTH: \( L = \theta r \) (in radians)
AREA of SECTOR: \( A = \frac{1}{2} \theta r^2 \)

Both in formula booklet

3.2 - UNIT CIRCLE

UNIT CIRCLE ⇒ Defined as a circle with its center at the origin and with a radius of 1.

RELATIONSHIP WITH SIN/COS ⇒

Looking at the diagram below, we can analyse the right-angled triangle created by the point \( P(a, b) \).

Using SOHCAHTOA:
\[
\sin \theta = \frac{b}{r} = \frac{b}{1} = b \\
\cos \theta = \frac{a}{r} = a \\
\]

So \( a = \cos \theta \)
\( b = \sin \theta \)
or \( P(a, b) \Rightarrow P(\cos \theta, \sin \theta) \)

SPECIFIC VALUES ⇒

By learning the coordinates on the unit circle at given angles, you can learn important values of \( \sin \theta \) and \( \cos \theta \).

The diagram to the right shows specific values with multiples of \( \frac{\pi}{4} \) & \( \frac{\pi}{6} \).
As a full circle is 2\( \pi \), these values repeat every 2\( \pi \).
3.3 - IDENTITIES

TRIGONOMETRIC

IDENTITY ⇒ Looking back to the triangle in the unit circle:
\[ \tan \theta = \frac{\text{opp}}{\text{adj}} \quad \Rightarrow \quad \tan \theta = \frac{\sin\theta}{\cos\theta} \]

E.G. ⇒ Calculate \( \frac{\sin\pi}{\cos\pi} \):
\[ \frac{\sin\pi}{\cos\pi} = \tan\pi = 0 \]

E.G. ⇒ Simplify \( 3\sin x + 2\cos x \tan x \):
\[ 3\sin x + 2\cos x \left( \frac{\sin x}{\cos x} \right) = 3\sin x + 2\sin x = 5\sin x \]

PYTHAGOREAN

IDENTITY ⇒ Using the pythagorean theorem on triangle from the unit circle, we get:
\[ a^2 + b^2 = 1^2 \quad \Rightarrow \quad \cos^2 \theta + \sin^2 \theta = 1 \]

E.G. ⇒ Simplify \( \cos^2 \theta \sin^3 \theta + \sin^3 \theta \):
\[ \text{As} \quad \cos^2 \theta \sin^3 \theta + \sin^3 \theta = \sin \theta (\cos^2 \theta + \sin^2 \theta) \]
We get \( \sin \theta (1) = \sin \theta \)

E.G. ⇒ Simplify \( 3\sin^2 \theta + 3\cos^2 \theta = 3(\sin^2 \theta + \cos^2 \theta) = 3(1) = 3 \)

DOUBLE ANGLE

FORMULAe ⇒ We also have formulae for \( \sin 2\theta \) & \( \cos 2\theta \):

\[ \sin 2\theta = 2\sin \theta \cos \theta \]
\[ \cos 2\theta = \cos^2 \theta - \sin^2 \theta \]
\[ \text{or} \quad = 1 - 2\sin^2 \theta \]
\[ \text{or} \quad = 2\cos^2 \theta - 1 \]

E.G. ⇒ If \( \sin \theta = \frac{4}{5} \) & \( \cos \theta = \frac{3}{5} \):

a) FIND \( \sin 2\theta \):
\[ \sin 2\theta = 2\sin \theta \cos \theta \]
\[ = 2 \left( \frac{4}{5} \right) \left( \frac{3}{5} \right) = \frac{24}{25} \]

b) FIND \( \cos 2\theta \):
\[ \cos 2\theta = 1 - 2\sin^2 \theta \]
\[ = 1 - 2 \left( \frac{4}{5} \right)^2 = 1 - \frac{32}{25} = \frac{-7}{25} \]
3.4 - TRIG. FUNCTION FEATURES

**TERMINOLOGY**
- **Principal Axis**
- **Amplitude**
- **Period**

**SINE CURVE**
- $y = \sin x$
- Values at $\pm 1$
- Key angles:
  - $90^\circ$: $\frac{\pi}{2}$
  - $180^\circ$: $\pi$
  - $270^\circ$: $\frac{3\pi}{2}$

**COSINE CURVE**
- $y = \cos x$
- Values at $\pm 1$
- Key angles:
  - $0^\circ$: $0$
  - $90^\circ$: $\frac{\pi}{2}$
  - $180^\circ$: $\pi$

**TAN**
- $y = \tan x$
- Key angles:
  - $90^\circ$: $\frac{\pi}{2}$
  - $180^\circ$: $\pi$

**TRANSFORMATIONS**

- **Sine**
  - $y = a \sin (b(x-c)) + d$
  - Principal axis at $y=d$
  - Period is $\frac{2\pi}{|b|}$
  - Amplitude is $|a|$
  - Horiz. translation by $c$

- **Tangent**
  - $y = a \tan (b(x-c)) + d$
  - Pr. axis at $y=d$
  - Period is $\frac{\pi}{|b|}$
  - Amplitude undefined
  - Horiz. trans. by $c$
3.5 - SOLVING TRIG. EQUATIONS

METHOD 1: Using Graphs

E.G. ⇒ Solve \( \sin(2x) = 0.7 \)
for \( 0 \leq x \leq 2 \): Use graph, we see \( x \approx 0.4 \) & 1.2

METHOD 2: Using technology

E.G. ⇒ Solve \( \sin(2x) = 0.7 \)
for \( 0 \leq x \leq 2 \):
There are a couple of ways with GDC:

a) Enter in 2 lines ⇒ \( Y_1 = \sin 2x \) & \( Y_2 = 0.7 \)
   ⇒ Find intersection ⇒ \( x = 0.3877 \) & 1.1731

b) Rearrange to get \( x = \frac{\sin^{-1}(0.7)}{2} \) and use
   GDC to calculate this value.

METHOD 3: Using algebraic methods

E.G. ⇒ Solve \( \sin(2x) = -\frac{1}{2} \)
for \( 0 \leq x \leq 2\pi \)
We will need to observe the unit circle
for when the y-coordinate is \(-\frac{1}{2}\).

We must make an adjustment as we are looking at \( 2x \). So the
range of angles doubles, to \( 0 \rightarrow 4\pi \). From the unit circle,
\( \sin \theta = -\frac{1}{2} \) at \( \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6} \). We must halve these to reach
our answers for \( x : \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12} \).

3.6 - TRIANGLE TRIG.

FOR TRIANGLES:

AREA RULE: \( \text{Area} = \frac{1}{2} \text{ab \sin C} \)

SINE RULE: \( \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \)

COSINE RULE: \( a^2 = b^2 + c^2 - 2bc \cos A \)
   or \( \cos A = \frac{b^2 + c^2 - a^2}{2bc} \)