2.3 The binomial theorem [55 marks]

The values in the fourth row of Pascal’s triangle are shown in the following table.

\[
\begin{array}{cccccc}
1 & 4 & 6 & 4 & 1 \\
\end{array}
\]

1a. Write down the values in the fifth row of Pascal’s triangle. [2 marks]
Hence or otherwise, find the term in $x^3$ in the expansion of $(2x + 3)^5$. [5 marks]

2. The third term in the expansion of $(2x + p)^6$ is $60x^4$. Find the possible values of $p$. [7 marks]

Consider the expansion of 
\[
(2x^3 + \frac{b}{x})^8 = 256x^{24} + 3072x^{20} + \ldots + kx^0 + \ldots
\]

3a. Find $b$. [3 marks]

3b. Find $k$. [3 marks]

Consider the expansion of 
\[
(3x^2 + 2)^9.
\]

4a. Write down the number of terms in the expansion. [1 mark]

4b. Find the term in $x^4$. [5 marks]

5a. Expand $(2 + x)^4$ and simplify your result. [3 marks]

5b. Hence, find the term in $x^2$ in 
\[
(2 + x)^4 \left(1 + \frac{1}{x^2}\right).
\]
6. Find the term in $x^4$ in the expansion of $\left(3x^2 - \frac{2}{x}\right)^5$. [6 marks]

The fifth term in the expansion of the binomial $(a + b)^n$ is given by $\binom{n}{4} a^n b^4$. [1 mark]

7a. Write down the value of \( n \). [2 marks]

7b. Write down \( a \) and \( b \), in terms of \( p \) and/or \( q \). [3 marks]

7c. Write down an expression for the sixth term in the expansion. [3 marks]

8a. Expand $(x - 2)^4$ and simplify your result. [3 marks]

8b. Find the term in $x^3$ in $(3x + 4)(x - 2)^4$. [3 marks]

9. Find the term $x^3$ in the expansion of $\left(\frac{2}{x}x - 3\right)^8$. [5 marks]