Basic logarithms [57 marks]

1a. Find $\log_2{32}$.

**Markscheme**

5  A1  N1

[1 mark]

1b. Given that $\log_2 \left( \frac{32^x}{8^y} \right)$ can be written as $px + qy$, find the value of $p$ and of $q$.

**Markscheme**

METHOD 1

$\log_2 \left( \frac{32^x}{8^y} \right) = \log_2{32^x} - \log_2{8^y}$ (A1)

$= x\log_2{32} - y\log_2{8}$ (A1)

$\log_2{8} = 3$ (A1)

$p = 5$, $q = -3$ (accept $5x - 3y$)  A1  N3

METHOD 2

$\frac{32^x}{8^y} = \left( \frac{2^5}{2^3} \right)^y$ (A1)

$= \frac{2^x}{2^y}$ (A1)

$= \frac{2^{5x-3y}}{2^y}$ (A1)

$\log_2{\frac{2^{5x-3y}}{2^y}} = 5x - 3y$

$p = 5$, $q = -3$ (accept $5x - 3y$)  A1  N3

[4 marks]
An arithmetic sequence has the first term \( \ln a \) and a common difference \( \ln 3 \).
The 13th term in the sequence is \( 8 \ln 9 \). Find the value of 
\[ a. \]

Markscheme

Note: There are many approaches to this question, and the steps may be done in any order. There are 3 relationships they may need to apply at some stage, for the 3rd, 4th and 5th marks. These are

- equating bases \( \text{eg recognising } 9 = 3^2 \)
- \log \text{ rules: } \ln b + \ln c = \ln(bc), \ln b - \ln c = \ln \left( \frac{b}{c} \right) \)
- exponent rule: \( \ln b^n = n \ln b \)

The exception to the \textit{FT} rule applies here, so that if they demonstrate correct application of the 3 relationships, they may be awarded the \( A \) marks, even if they have made a previous error. However all applications of a relationship need to be correct. Once an error has been made, do not award \( A1FT \) for their final answer, even if it follows from their working.

Please check working and award marks in line with the mark scheme.

correct substitution into \( u_{13} \) formula \( (A1) \)
\[ \text{eg } \ln a + (13 - 1) \ln 3 \]
set up equation for \( u_{13} \) in any form (seen anywhere) \( (M1) \)
\[ \text{eg } \ln a + 12 \ln 3 = 8 \ln 9 \]
correct application of relationships \( (A1)(A1)(A1) \)
\[ a = 81 \quad \text{A1} \quad \text{N3} \]

[6 marks]

Examples of application of relationships

Example 1

correct application of exponent rule for logs \( (A1) \)
\[ \text{eg } \ln a + \ln 3^2 = \ln 9^8 \]
correct application of addition rule for logs \( (A1) \)
\[ \text{eg } \ln (a^3 + 12) = \ln 9^8 \]
substituting for 9 or 3 in \( \ln \) expression in equation \( (A1) \)
\[ \text{eg } \ln (a^3 + 12) = \ln 3^2, \ln (a^3 + 9^3) = \ln 9^8 \]

Example 2
recognising \( 9 = 3^2 \) \( (A1) \)
\[ \text{eg } \ln a + 12 \ln 3 = 8 \ln 3^2, \ln a + 12 \ln 9^\frac{1}{2} = 8 \ln 9 \]
one correct application of exponent rule for logs relating \( \ln 9 \) to \( \ln 3 \) \( (A1) \)
\[ \text{eg } \ln a + 12 \ln 3 = 16 \ln 3, \ln a + 6 \ln 9 = 8 \ln 9 \]

another correct application of exponent rule for logs \( (A1) \)
\[ \text{eg } \ln a = \ln 3^4, \ln a = \ln 9^2 \]
3a. Given that \(2^m = 8\) and \(2^n = 16\), write down the value of \(m\) and of \(n\). [2 marks]

Markscheme
\[ m = 3, \ n = 4 \quad A1A1 \quad N2 \]
[2 marks]

3b. Hence or otherwise solve \(8^{2^{x+1}} = 16^{2^{x-3}}\). [4 marks]

Markscheme
attempt to apply \((2^x)^3 = 2^{3x}\) (M1)
\[ eg \ 6x + 3, \ 4(2x - 3) \]
equating their powers of 2 (seen anywhere) (M1)
\[ eg \ 3(2x + 1) = 8x - 12 \]
correct working \( A1 \)
\[ eg \ 8x - 12 = 6x + 3, \ 2x = 15 \]
\[ x = \frac{15}{2} \quad (7.5) \quad A1 \quad N2 \]
[4 marks]
Total [6 marks]
Write the expression \(3 \ln 2 - \ln 4\) in the form \(\ln k\), where \(k \in \mathbb{Z}\).

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**Markscheme**

correct application of \(\ln a^b = b \ln a\) (seen anywhere) \(\text{(A1)}\)

eg \(\ln 4 = 2 \ln 2, 3 \ln 2 = \ln 2^3, 3 \log 2 = \log 8\)
correct working \(\text{(A1)}\)

eg \(3 \ln 2 - 2 \ln 2, \ln 8 - \ln 4\)
\(\ln 2\) (accept \(k = 2\)) \(\text{A1} \text{ N2} \)

[3 marks]

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Hence or otherwise, solve \(3 \ln 2 - \ln 4 = -\ln x\).

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**Markscheme**

**METHOD 1**

try to substitute their answer into the equation \(\text{(M1)}\)

eg \(\ln 2 = -\ln x\)
correct application of a log rule \(\text{(A1)}\)

eg \(\ln \frac{1}{2}, \ln \frac{x}{2} = \ln x, \ln 2 + \ln x = \ln 2x \quad (= 0)\)
\(x = \frac{1}{2} \text{ A1 N2} \)

**METHOD 2**

try to rearrange equation, with \(3 \ln 2\) written as \(\ln 2^3\) or \(\ln 8\) \(\text{(M1)}\)

eg \(\ln x = \ln 4 - \ln 2^3, \ln 8 + \ln x = \ln 4, \ln 2^3 = \ln 4 - \ln x\)
correct working applying \(\ln a \pm \ln b\) \(\text{(A1)}\)

eg \(\frac{1}{2}, 8x = 4, \ln 2^3 = \ln \frac{4}{x}\)
\(x = \frac{1}{2} \text{ A1 N2} \)

[3 marks]

*Total [6 marks]*
Write down the value of

5a. \( \log_3{27} \):

**Markscheme**

(i) \( \log_3{27} = 3 \) \( A1 \) \( N1 \)

[1 mark]

5b. \( \log_{\frac{1}{8}}{1} \):

**Markscheme**

(ii) \( \log_{\frac{1}{8}}{1} = -1 \) \( A1 \) \( N1 \)

[1 mark]

5c. \( \log_{16}{4} \):

**Markscheme**

(iii) \( \log_{16}{4} = \frac{1}{2} \) \( A1 \) \( N1 \)

[1 mark]

5d. Hence, solve

\[ \log_3{27} + \log_{\frac{1}{8}}{1} - \log_{16}{4} = \log{x}. \]

**Markscheme**

Correct equation with their three values \( (A1) \)

eg

\[ \frac{3}{2} = \log_{4}{x}, \; 3 + (-1) - \frac{1}{2} = \log_{4}{x} \]

Correct working involving powers \( (A1) \)

eg

\[ x = 4^{\frac{3}{2}}, \; 4^{\frac{3}{2}} = 4^{\log_{4}{x}} \]

\[ x = 8 \; A1 \) \( N2 \]

[3 marks]

Find the value of each of the following, giving your answer as an integer.

6a. \( \log_{6}{36} \):

[2 marks]
Markscheme

correct approach  \((A1)\)

\(eg\)

\(6^2 = 36, \ 6^2\)

2 \(A1\ \ N2\)

[2 marks]

6b. \(\log_{6}4 + \log_{6}9\)  \([2\ marks]\)

Markscheme

correct simplification  \((A1)\)

\(eg\)

\(\log_{6}36, \ \log(4 \times 9)\)

2 \(A1\ \ N2\)

[2 marks]

6c. \(\log_{6}2 - \log_{6}12\)  \([3\ marks]\)

Markscheme

correct simplification  \((A1)\)

\(eg\)

\(\log_{6}\frac{2}{17}, \ \log(2 \div 12)\)

correct working  \((A1)\)

\(eg\)

\(\log_{6}\frac{1}{3}, \ 6^{-1} = \frac{1}{6}, \ 6^x = \frac{1}{6}\)

\(-1 \ A1\ \ N2\)

[3 marks]

7a. Find the value of

\(\log_{2}40 - \log_{2}5\).  \([3\ marks]\)

Markscheme

evidence of correct formula  \((M1)\)

\(eg\)

\(\log a - \log b = \log \frac{a}{b}\),

\(\log \left(\frac{40}{5}\right), \ \ log 8 + \log 5 - \log 5\)

**Note:** Ignore missing or incorrect base.

correct working  \((A1)\)

\(eg\)

\(\log_{2}8, \ 2^3 = 8\)

\(\log_{2}40 - \log_{2}5 = 3 \ A1\ \ N2\)

[3 marks]
Markscheme

attempt to write
8 as a power of
2 (seen anywhere)  \((M1)\)

\[eg\]
\((2^3)^{\log_5 2}\),
\(2^3 = 8\),
\(2^1\)

multiplying powers \((M1)\)

\[eg\]
\(2^{10\log_5 3}\),
\(a \log_5 3\)

correct working \((A1)\)

\[eg\]
\(2^{10\log_5 125}\),
\(\log_5 5^3\),
\((2^{\log_5 5})^3\)

\(8^{\log_5 5} = 125\)  \(A1\)  \(N3\)

[4 marks]

Let
\(\log_3 p = 6\) and
\(\log_3 q = 7\).

8a. (a) Find
\(\log_3 p^2\).

(b) Find
\(\log_3 \left(\frac{2}{4}\right)\).

(c) Find
\(\log_3 (9p)\).

Markscheme

(a)  METHOD 1
evidence of correct formula \((M1)\)

\[eg\]
\(\log u^n = n \log u\),
\(2 \log_3 p\)

\(\log_3 (p^2) = 12\)  \(A1\)  \(N2\)

METHOD 2
valid method using
\(p = 3^6\)  \((M1)\)

\[eg\]
\(\log_3 (3^6)^2\),
\(\log 3^{12}\),
\(12 \log_3 3\)

\(\log_3 (p^2) = 12\)  \(A1\)  \(N2\)

[2 marks]
(b) **METHOD 1**
evidence of correct formula \((M1)\)

\[
\log\left(\frac{p}{q}\right) = \log p - \log q, \\
6 - 7 \\
\log_3\left(\frac{p}{q}\right) = -1 \quad \text{A1 N2}
\]

**METHOD 2**
valid method using
\(p = 3^6\) and
\(q = 3^7\) \((M1)\)

\[
\log_3\left(\frac{p}{q}\right) , \\
\log 3^{-1} , \\
-\log_3 3 \\
\log_3\left(\frac{p}{q}\right) = -1 \quad \text{A1 N2}
\]

[2 marks]

(c) **METHOD 1**
evidence of correct formula \((M1)\)

\[
\log_3 uv = \log_3 u + \log_3 v, \\
\log 9 + \log p \\
\log_3 9 = 2 \text{ (may be seen in expression)} \quad \text{A1}
\]

\[
2 + \log p \\
\log_3(9p) = 8 \quad \text{A1 N2}
\]

**METHOD 2**
valid method using
\(p = 3^6\) \((M1)\)

\[
\log_3(9 \times 3^6), \\
\log_3(3^2 \times 3^6) \\
correct working \quad \text{A1}
\]

\[
\log_3 9 + \log_3 3^6, \\
\log_3 3^8 \\
\log_3(9p) = 8 \quad \text{A1 N2}
\]

[3 marks]

Total [7 marks]

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8b. Find
\[
\log_3 p^2.
\]

[2 marks]
**Markscheme**

**METHOD 1**

evidence of correct formula \( (M1) \)

eg

\[
\log u^n = n \log u ,
\]

\[
2 \log_3 p
\]

\( \log_3(p^2) = 12 \quad A1 \quad N2 \)

**METHOD 2**

valid method using

\( p = 3^6 \quad (M1) \)

eg

\[
\log_3(3^6)^2 ,
\]

\[
\log 3^{12} ,
\]

\[
12 \log_3 3
\]

\( \log_3(p^2) = 12 \quad A1 \quad N2 \)

[2 marks]

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8c Find

\( \log_3 \left( \frac{p}{q} \right) \).

**Markscheme**

**METHOD 1**

evidence of correct formula \( (M1) \)

eg

\[
\log \left( \frac{p}{q} \right) = \log p - \log q ,
\]

\( 6 - 7 \)

\( \log_3 \left( \frac{p}{q} \right) = -1 \quad A1 \quad N2 \)

**METHOD 2**

valid method using

\( p = 3^6 \) and

\( q = 3^1 \quad (M1) \)

eg

\[
\log_3 \left( \frac{3^6}{3^1} \right) ,
\]

\[
\log 3^{-1} ,
\]

\( - \log_3 3 \)

\( \log_3 \left( \frac{p}{q} \right) = -1 \quad A1 \quad N2 \)

[2 marks]

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8d Find

\( \log_3(9p) \).

**Markscheme**

**METHOD 1**

evidence of correct formula \( (M1) \)

eg

\[
\log (9p) = \log 9 + \log p ,
\]

\( 2 + \log_3 3 \)

\( \log_3 (9p) = 2 \quad A1 \quad N2 \)

**METHOD 2**

valid method using

\( p = 3^6 \) and

\( q = 3^1 \quad (M1) \)

eg

\[
\log_3 \left( \frac{3^6}{3^1} \right) ,
\]

\[
\log 3^{-1} ,
\]

\( - \log_3 3 \)

\( \log_3 (9p) = 2 \quad A1 \quad N2 \)

[3 marks]
Markscheme

METHOD 1

evidence of correct formula \( (M1) \)

\[ \begin{align*}
\log_3 uv &= \log_3 u + \log_3 v , \\
\log 9 + \log p \\
\log_3 9 &= 2 \text{ (may be seen in expression) } \quad A1 \\
2 + \log p \\
\log_3 (9p) &= 8 \quad A1 \quad N2
\end{align*} \]

METHOD 2

valid method using \( p = 3^6 \) \( (M1) \)

\[ \begin{align*}
\log_3 (9 \times 3^6) , \\
\log_3 (3^2 \times 3^6) \\
correct \ working \quad A1 \\
\log_3 9 + \log_3 3^6 , \\
\log_3 3^8 \\
\log_3 (9p) &= 8 \quad A1 \quad N2
\end{align*} \]

[3 marks]

Total [7 marks]