Sequences and Sums [141 marks]

The first three terms of a geometric sequence are \( u_1 = 0.64 \), \( u_2 = 1.6 \), and \( u_3 = 4 \).

1a. Find the value of \( r \). 

**Markscheme**

valid approach \((M1)\)

\[ \frac{u_2}{u_1} = \frac{4}{1.6} = r(0.64) \]

\( r = 2.5 \left( = \frac{5}{2} \right) \) \( A1 \) \( N2 \)  

[2 marks]

1b. Find the value of \( S_6 \). 

**Markscheme**

correct substitution into \( S_n \) \((A1)\)

\[ S_6 = \frac{0.64(2.5^n - 1)}{2.5 - 1} \]

\( S_6 = 103.74 \) (exact), 104 \( A1 \) \( N2 \)  

[2 marks]

1c. Find the least value of \( n \) such that \( S_n > 75000 \). 

**Markscheme**

METHOD 1 (analytic)

valid approach \((M1)\)

\[ \frac{0.64(2.5^n - 1)}{2.5 - 1} > 75000, \quad \frac{0.64(2.5^n - 1)}{2.5 - 1} = 75000 \]

correct inequality (accept equation) \((A1)\)

\[ n > 13.1803, \quad n = 13.2 \]

\( n = 14 \) \( A1 \) \( N1 \)

METHOD 2 (table of values)

both crossover values \((A2)\)

\[ S_{13} = 63577.8, \quad S_{14} = 158945 \]

\( n = 14 \) \( A1 \) \( N1 \)  

[3 marks]

Total [7 marks]

In an arithmetic sequence \( u_{10} = 8 \), \( u_{11} = 6.5 \).

2a. Write down the value of the common difference.  

[1 mark]
Markscheme

\[ d = -1.5 \quad A1 \quad N1 \]

[1 mark]

2b. Find the first term.

Markscheme

METHOD 1
valid approach \((M1)\)
\[ u_{10} = u_1 + 9d, \quad 8 = u_1 - 9(-1.5) \]
correct working \((A1)\)
\[ 8 = u_1 + 9d, \quad 6.5 = u_1 + 10d, \quad u_1 = 8 - 9(-1.5) \]
\[ u_1 = 21.5 \quad A1 \quad N2 \]

METHOD 2
attempt to list 3 or more terms in either direction \((M1)\)
\[ 9.5, 11, 12.5, \ldots ; 5, 3.5, 2, \ldots \]
correct list of 4 or more terms in correct direction \((A1)\)
\[ 9.5, 11, 12.5, 14 \]
\[ u_1 = 21.5 \quad A1 \quad N2 \]

[3 marks]

2c. Find the sum of the first 50 terms of the sequence.

Markscheme

correct expression \((A1)\)
\[ \begin{align*}
\text{eg } &\frac{50}{2}(2(21.5) + 49(-1.5)), \quad \frac{50}{2}(21.5 - 52), \\
\sum_{k=1}^{50} (12.5 + (k - 1)(-1.5))
\end{align*} \]
sum = \(-762.5\) (exact) \quad A1 \quad N2

[2 marks]

Total [6 marks]

3. Ramiro walks to work each morning. During the first minute he walks 80 metres. In each subsequent minute he walks 90\% of the distance walked during the previous minute.

The distance between his house and work is 660 metres. Ramiro leaves his house at 08:00 and has to be at work by 08:15.

Explain why he will not be at work on time.

Markscheme

METHOD 1
recognize that the distance walked each minute is a geometric sequence \((M1)\)
\[ r = 0.9, \quad \text{valid use of } 0.9 \]
recognize that total distance walked is the sum of a geometric sequence \((M1)\)
\[ S_n, \quad a \left( \frac{1-r^n}{1-r} \right) \]
correct substitution into the sum of a geometric sequence \((A1)\)

\[80 \left( \frac{1 - 0.9^n}{1 - 0.9} \right)\]

any correct equation with sum of a geometric sequence \((A1)\)

\[80 \left( \frac{0.9^n - 1}{0.9 - 1} \right) = 660, \quad 1 - 0.9^n = \frac{66}{80}\]

attempt to solve their equation involving the sum of a GP \((M1)\)

\[eg \text{ graph, algebraic approach}\]

\[n = 16.54290788 \quad A1\]

since \(n > 15\) \(R1\)

he will be late \(AG \quad N0\)

\[\text{Note: Do not award the } R \text{ mark without the preceding } A \text{ mark.}\]

**METHOD 2**

recognize that the distance walked each minute is a geometric sequence \((M1)\)

\[\text{eg } r = 0.9, \text{ valid use of } 0.9\]

recognize that total distance walked is the sum of a geometric sequence \((M1)\)

\[\text{eg } S_n, \quad a \left( \frac{1 - r^n}{1 - r} \right)\]

correct substitution into the sum of a geometric sequence \((A1)\)

\[80 \left( \frac{1 - 0.9^n}{1 - 0.9} \right)\]

attempt to substitute \(n = 15\) into sum of a geometric sequence \((M1)\)

\[\text{eg } S_{15}\]

correct substitution \((A1)\)

\[\text{eg } 80 \left( \frac{0.9^{15} - 1}{0.9 - 1} \right)\]

\[S_{15} = 635.287 \quad A1\]

since \(S < 660\) \(R1\)

he will not be there on time \(AG \quad N0\)

\[\text{Note: Do not award the } R \text{ mark without the preceding } A \text{ mark.}\]

**METHOD 3**

recognize that the distance walked each minute is a geometric sequence \((M1)\)

\[\text{eg } r = 0.9, \text{ valid use of } 0.9\]

recognize that total distance walked is the sum of a geometric sequence \((M1)\)

\[\text{eg } S_n, \quad a \left( \frac{1 - r^n}{1 - r} \right)\]

listing at least 5 correct terms of the GP \((M1)\)

15 correct terms \(A1\)

80, 72, 64.8, 58.32, 52.488, 47.2392, 42.5152, 38.2637, 34.4373, 30.9936, 27.8942, 25.1048, 22.59436, 20.3349, 18.3014

attempt to find the sum of the terms \((M1)\)

\[\text{eg } S_{15}, \quad 80 + 72 + 64.8 + 58.32 + 52.488 + \ldots + 18.301433\]

\[S_{15} = 635.287 \quad A1\]

since \(S < 660\) \(R1\)

he will not be there on time \(AG \quad N0\)

\[\text{Note: Do not award the } R \text{ mark without the preceding } A \text{ mark.}\]

[7 marks]
In an arithmetic sequence, the first term is 2 and the second term is 5.

4a. Find the common difference. [2 marks]

**Markscheme**
- correct approach (A1)
- eg $d = u_2 - u_1$, 5 - 2
- $d = 3$ A1 N2

2 marks

4b. Find the eighth term. [2 marks]

**Markscheme**
- correct approach (A1)
- eg $u_8 = 2 + 7 	imes 3$, listing terms
- $u_8 = 23$ A1 N2

2 marks

4c. Find the sum of the first eight terms of the sequence. [2 marks]

**Markscheme**
- correct approach (A1)
- eg $S_8 = \frac{8}{2}(2 + 23)$, listing terms, $\frac{8}{2}(2(2) + 7(3))$
- $S_8 = 100$ A1 N2

2 marks

**Total [6 marks]**

The first two terms of a geometric sequence $u_n$ are $u_1 = 4$ and $u_2 = 4.2$.

5a. (i) Find the common ratio. [5 marks]

(ii) Hence or otherwise, find $u_5$. 
Markscheme

(i) valid approach \((M1)\)

\[ r = \frac{u_2}{u_1} = \frac{4}{1.2} \]

\[ r = 1.05 \text{ (exact) } \quad A1 \quad N2 \]

(ii) attempt to substitute into formula, with their \(r\) \((M1)\)

\[ 4 \times 1.05^n, \quad 4 \times 1.05 \times 1.05 \ldots \]

correct substitution \((A1)\)

\[ 4 \times 1.05^4, \quad 4 \times 1.05 \times 1.05 \times 1.05 \times 1.05 \]

\[ u_5 = 4.862025 \text{ (exact), } 4.86 [4.86, 4.87] \quad A1 \quad N2 \]

[5 marks]

5b. Another sequence \(v_n\) is defined by \(v_n = an^k\), where \(a, k \in \mathbb{R}\), and \(n \in \mathbb{Z}^+\), such that \(v_1 = 0.05\) and \(v_2 = 0.25\).

(i) Find the value of \(a\).

(ii) Find the value of \(k\).

Markscheme

(i) attempt to substitute \(n = 1\) \((M1)\)

\[ 0.05 = a \times 1^k \]

\[ a = 0.05 \quad A1 \quad N2 \]

(ii) correct substitution of \(n = 2\) into \(v_2\) \(A1\)

\[ 0.25 = a \times 2^k \]

correct work \((A1)\)

\[ \text{finding intersection point, } k = \log_2 \left( \frac{0.25}{0.05} \right) = \frac{\log_5}{\log_2} \]

\[ 2.32192 \]

\[ k = \log_5 \text{ (exact), } 2.32 [2.32, 2.33] \quad A1 \quad N2 \]

[5 marks]

5c. Find the smallest value of \(n\) for which \(v_n > u_n\).
The sides of a square are 16 cm in length. The midpoints of the sides of this square are joined to form a new square and four triangles (diagram 1). The process is repeated twice, as shown in diagrams 2 and 3.

Let $x_n$ denote the length of one of the equal sides of each new triangle.

Let $A_n$ denote the area of each new triangle.

The following table gives the values of $x_n$ and $A_n$, for $1 \leq n \leq 3$. Copy and complete the table. (Do not write on this page.)

<table>
<thead>
<tr>
<th>$n$</th>
<th>$x_n$</th>
<th>$A_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>32</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Markscheme**

valid method for finding side length \((M1)\)

\[
8^2 + 8^2 = c^2, \ 45 - 45 - 90 \text{ side ratios, } 8\sqrt{2}, \ \frac{1}{2}s^2 = 16, \ x^2 + x^2 = 8^2
\]

correct working for area \((A1)\)

\[
\frac{1}{2} \times 4 \times 4
\]

<table>
<thead>
<tr>
<th>(n)</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_n)</td>
<td>8</td>
<td>(\sqrt{32})</td>
<td>4</td>
</tr>
<tr>
<td>(A_n)</td>
<td>32</td>
<td>16</td>
<td>8</td>
</tr>
</tbody>
</table>

\((A1A1) \ N2N2 \)

[4 marks]

The process described above is repeated. Find \(A_6\). \([4 \text{ marks}]\)

**Markscheme**

**METHOD 1**

recognize geometric progression for \(A_n\) \((R1)\)

\[
u_n = u_1 r^{n-1}
\]

\[r = \frac{1}{2} \quad \text{ (A1)}
\]

correct working \((A1)\)

\[
32 \left(\frac{1}{2}\right)^5; \ 4, 2, 1, \frac{1}{2}, \frac{1}{4}, \ldots
\]

\[A_6 = 1 \quad A1 \ N3\]

**METHOD 2**

attempt to find \(x_6\) \((M1)\)

\[
8 \left(\frac{1}{\sqrt{2}}\right)^5; \ 2\sqrt{2}, 2, \sqrt{2}, 1, \ldots
\]

\[x_6 = \sqrt{2} \quad \text{(A1)}
\]

correct working \((A1)\)

\[
\frac{1}{2} \left(\sqrt{2}\right)^2
\]

\[A_6 = 1 \quad A1 \ N3\]

[4 marks]

6c. Consider an initial square of side length \(k\) cm. The process described above is repeated indefinitely. The total area of the shaded regions is \(k\) cm\(^2\). Find the value of \(k\). \([7 \text{ marks}]\)
Markscheme

METHOD 1
recognize infinite geometric series \((R1)\)
\begin{align*}
&eg \\
&S_n = \frac{a}{1-r}, \quad |r| < 1
\end{align*}
area of first triangle in terms of \(k\) \((A1)\)
\begin{align*}
&eg \\
&\frac{1}{2} \left( \frac{k}{2} \right)^2
\end{align*}
attempt to substitute into sum of infinite geometric series (must have \(k\)) \((M1)\)
\begin{align*}
&eg \quad \frac{\frac{1}{4} \left( \frac{k}{2} \right)^2}{1 - \frac{k}{2}} \\
&\text{correct equation } A1 \quad \frac{k}{1 - \frac{k}{2}}
\end{align*}
correct working \((A1)\)
\begin{align*}
&eg \\
&k^3 = 4k \\
&\text{valid attempt to solve their quadratic } (M1)
\end{align*}
\begin{align*}
&eg \\
&k(k - 4), \quad k = 4 \text{ or } k = 0 \quad A1 \quad N2
\end{align*}

METHOD 2
recognizing that there are four sets of infinitely shaded regions with equal area \(R1\)
area of original square is \(k^2\) \((A1)\)
so total shaded area is \(\frac{k^2}{4}\) \((A1)\)
correct equation \(\frac{k^2}{4} = k \quad A1\)
\(k^2 = 4k \quad A1 \quad N2\)
valid attempt to solve their quadratic \((M1)\)
\begin{align*}
&eg \\
&k(k - 4), \quad k = 4 \text{ or } k = 0 \quad A1 \quad N2
\end{align*}

[7 marks]

The sums of the terms of a sequence follow the pattern
\[S_1 = 1 + k, \quad S_2 = 5 + 3k, \quad S_3 = 12 + 7k, \quad S_4 = 22 + 15k, \ldots, \text{ where } k \in \mathbb{Z}.
\]

7a. Given that
\begin{align*}
&u_1 = 1 + k, \quad \text{find} \\
u_2, \quad u_3 \quad \text{and} \\
u_4.
\end{align*}

[4 marks]
**Markscheme**

valid method  \((M1)\)

\[ u_2 = S_2 - S_1, \quad 1 + k + u_2 = 5 + 3k \]

\[ u_2 = 4 + 2k, \quad u_3 = 7 + 4k, \quad u_4 = 10 + 8k \]

\(A1A1A1 \quad N4\)  

[4 marks]

7b. Find a general expression for \(u_n\). 

**Markscheme**

correct AP or GP  \((A1)\)

\(eg\) finding common difference is 3, common ratio is 2

valid approach using arithmetic and geometric formulas  \((M1)\)

\[ 1 + 3(n - 1) \quad and \]

\[ r^{n-1}k \]

\[ u_n = 3n - 2 + 2^{n-1}k \quad A1A1 \quad N4\]

Note: Award \(A1\) for \(3n - 2\), \(A1\) for \(2^{n-1}k\).

[4 marks]

The first three terms of an infinite geometric sequence are \(m - 1, \ 6, \ m + 4\), where \(m \in \mathbb{Z}\).

8a. Write down an expression for the common ratio, \(r\). 

**Markscheme**

correct expression for \(r\)  \((A1)\)

\(eg\)

\[ r = \frac{6}{m-1}, \quad \frac{m+4}{6} \]

[2 marks]

8b. Hence, show that \(m\) satisfies the equation \(m^2 + 3m - 40 = 0\).
Markscheme

correct equation  \( A1 \)

eg
\[ \frac{6}{m+4} = \frac{m+4}{6} , \quad \frac{6}{m+4} = \frac{m+4}{6} \]

correct working  \((A1)\)

eg
\( (m + 4)(m - 1) = 36 \)

correct working  \( A1 \)

eg
\[ m^2 - m + 4m - 4 = 36 , \quad m^2 + 3m - 4 = 36 \]
\[ m^2 + 3m - 40 = 0 \quad AG \quad N0 \]

[2 marks]

8c. Find the two possible values of \( m \).

Markscheme

valid attempt to solve  \((M1)\)

eg
\( (m + 8)(m - 5) = 0 , \quad m = \frac{-3 \pm \sqrt{9 + 4 \times 40}}{2} \)
\[ m = -8 , \quad m = 5 \quad A1A1 \quad N3 \]

[3 marks]

8d. Find the possible values of \( r \).

Markscheme

attempt to substitute any value of \( m \) to find \( r \)  \((M1)\)

eg
\[ \frac{6}{m-1} , \quad \frac{5+4}{6} \]
\[ r = \frac{3}{2} , \quad r = -\frac{2}{3} \quad A1A1 \quad N3 \]

[3 marks]

8e. The sequence has a finite sum.

State which value of \( r \) leads to this sum and justify your answer.


### Markscheme

\[ r = -\frac{2}{3} \] \hspace{1cm} (may be seen in justification) \hspace{0.5cm} A1

valid reason \hspace{0.5cm} R1 \hspace{0.5cm} N0

eg

\[ |r| < 1, \quad -1 < -\frac{2}{3} < 1 \]

**Notes:** Award R1 for

\[ |r| < 1 \text{ only if } A1 \text{ awarded.} \]

[2 marks]

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**8f.** The sequence has a finite sum.

Calculate the sum of the sequence.

### Markscheme

finding the first term of the sequence which has \( |r| < 1 \) \hspace{0.5cm} (A1)

eg

\[ -8 - 1, 6 \div -\frac{2}{3} \]

\[ u_1 = -9 \text{ (may be seen in formula)} \hspace{0.5cm} (A1) \]

correct substitution of \( u_1 \) and their \( r \) into \( \frac{u_1}{1 - r} \), as long as \( |r| < 1 \) \hspace{0.5cm} A1

eg

\[ \begin{align*}
S_\infty &= \frac{-9}{1 - (-\frac{2}{3})} = \frac{-9}{\frac{5}{3}} \\
S_\infty &= -\frac{27}{5} \quad (= -5.4) \quad A1 \hspace{0.5cm} N3
\end{align*} \]

[4 marks]

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**9a.** An arithmetic sequence is given by

5, 8, 11, ...

(a) Write down the value of \( d \). \hspace{5cm} [7 marks]

(b) Find

(i) \( u_{100} \).

(ii) \( S_{100} \).

(c) Given that \( u_n = 1502 \), find the value of \( n \).
Markscheme

(a) 
\[ d = 3 \quad A1 \quad N1 \]  
[1 mark]

(b) (i) correct substitution into term formula \((A1)\)  
e.g. 
\[ u_{100} = 5 + 3(99) \, , \]
\[ 5 + 3(100 - 1) \]
\[ u_{100} = 302 \quad A1 \quad N2 \]

(ii) correct substitution into sum formula \((A1)\)  
e.g. 
\[ S_{100} = \frac{100}{2} (2(5) + 99(3)) \, , \]
\[ S_{100} = \frac{100}{2} (5 + 302) \]
\[ S_{100} = 15350 \quad A1 \quad N2 \]  
[4 marks]

(c) correct substitution into term formula \((A1)\)  
e.g. 
\[ 1502 = 5 + 3(n - 1) \, , \]
\[ 1502 = 3n + 2 \]
\[ n = 500 \quad A1 \quad N2 \]  
[2 marks]

Total [7 marks]

9b. Write down the value of \(d\).  
[1 mark]

Markscheme

\[ d = 3 \quad A1 \quad N1 \]  
[1 mark]

9c. Find  
\[ u_{100} ; \]  
\[ S_{100} ; \]  
[4 marks]
Markscheme

(i) correct substitution into term formula \((A1)\)

\[ u_{100} = 5 + 3(99) , \]
\[ 5 + 3(100 - 1) \]
\[ u_{100} = 302 \quad A1 \quad N2 \]

(ii) correct substitution into sum formula \((A1)\)

\[ S_{100} = \frac{100}{2} (2(5) + 99(3)) , \]
\[ S_{100} = \frac{100}{2} (5 + 302) \]
\[ S_{100} = 15350 \quad A1 \quad N2 \]

[4 marks]

Given that \(u_n = 1502\), find the value of \(n\).

Markscheme

correct substitution into term formula \((A1)\)

\[ 1502 = 5 + 3(n - 1) , \]
\[ 1502 = 3n + 2 \]
\[ n = 500 \quad A1 \quad N2 \]

[2 marks]

Total [7 marks]

The sum of the first three terms of a geometric sequence is 62.755, and the sum of the infinite sequence is 440. Find the common ratio.
**Markscheme**

correct substitution into sum of a geometric sequence \( A1 \)

\( eg \)

\[
62.755 = u_1 \left( \frac{1 - r^3}{1 - r} \right),
\]

\( u_1 + u_1r + u_1r^2 = 62.755 \)

correct substitution into sum to infinity \( A1 \)

\( eg \)

\[
\frac{u_1}{1 - r} = 440
\]

attempt to eliminate one variable \( (M1) \)

\( eg \) substituting

\( u_1 = 440(1 - r) \)

correct equation in one variable \( (A1) \)

\( eg \)

\[
62.755 = 440(1 - r) \left( \frac{1 - r^3}{1 - r} \right),
\]

\[
440(1 - r)(1 + r + r^2) = 62.755
\]

evidence of attempting to solve the equation in a single variable \( (M1) \)

\( eg \) sketch, setting equation equal to zero,

\[
62.755 = 440(1 - r^3)
\]

\( r = 0.95 = \frac{19}{20} \) \( A1 \) \( N4 \)

[6 marks]

11a. The first three terms of an arithmetic sequence are 5, 6.7, 8.4.

Find the common difference.

**Markscheme**

valid method \( (M1) \)

\( eg \) subtracting terms, using sequence formula

\( d = 1.7 \) \( A1 \) \( N2 \)

[2 marks]

11b. The first three terms of an arithmetic sequence are 5, 6.7, 8.4.

Find the 28th term of the sequence.

**Markscheme**

correct substitution into term formula \( (A1) \)

\( eg \)

\[
5 + 27(1.7)
\]

28th term is 50.9 (exact) \( A1 \) \( N2 \)

[2 marks]
The first three terms of an arithmetic sequence are $5, 6.7, 8.4$.

Find the sum of the first 28 terms.

**Markscheme**

correct substitution into sum formula \((A1)\)

e.g.
$$S_{28} = \frac{28}{2}(2(5) + 27(1.7)),$$
$$\frac{28}{2}(5 + 50.9)$$
$$S_{28} = 782.6 \text{ (exact)}$$
$$782, 783 \ A1 \ N2$$

[2 marks]

The first three terms of an arithmetic sequence are 36, 40, 44,.....

12a. (i) Write down the value of $$d$$. \[3 marks\]

(ii) Find $$u_8$$.

**Markscheme**

(i) \(d = 4 \ A1 \ N1\)

(ii) evidence of valid approach \((M1)\)

e.g.
$$u_8 = 36 + 7(4), \text{ repeated addition of } d \text{ from 36}$$
$$u_8 = 64 \ A1 \ N2$$

[3 marks]

12b. (i) Show that $$S_n = 2n^2 + 34n$$. \[3 marks\]

(ii) Hence, write down the value of $$S_{14}$$.

**Markscheme**

(i) correct substitution into sum formula \(A1\)

e.g.
$$S_n = \frac{n}{2}\{2(36) + (n - 1)(4)\},$$
$$\frac{n}{2}\{72 + 4n - 4\}$$

evidence of simplifying

e.g.
$$\frac{n}{2}\{4n + 68\} \ A1$$
$$S_n = 2n^2 + 34n \ AG \ N0$$

(ii) \(868 \ A1 \ N1\)

[3 marks]
13a. The first term of a geometric sequence is 200 and the sum of the first four terms is 324.8. Find the common ratio. 

**Markscheme**

correct substitution into sum of a geometric sequence \((A1)\)
e.g.
\[200 \left(\frac{1 - r^4}{1 - r}\right),\]
\[200 + 200r + 200r^2 + 200r^3\]
try to set up an equation involving a sum and 324.8 \((M1)\)
e.g.
\[200 \left(\frac{1 - r^4}{1 - r}\right) = 324.8,\]
\[200 + 200r + 200r^2 + 200r^3 = 324.8\]
\[r = 0.4 \text{ (exact)} \quad (A2) \quad (N3)\]

[4 marks]

13b. The first term of a geometric sequence is 200 and the sum of the first four terms is 324.8. Find the tenth term.

**Markscheme**
correct substitution into formula \((A1)\)
e.g.
\[u_{10} = 200 \times 0.4^9\]
\[u_{10} = 0.0524288 \text{ (exact)},\]
\[0.0524 \quad (A1) \quad (N1)\]

[2 marks]

14a. In an arithmetic sequence, \(u_1 = 2\) and \(u_3 = 8\). Find \(d\).

**Markscheme**
attempt to find \(d\) \((M1)\)
e.g.
\[
\frac{8 - 2}{2} = 2 + 2d
\]
\[d = 3 \quad (A1) \quad (N2)\]

[2 marks]

14b. Find \(u_{20}\).

**Markscheme**
Markscheme

14c. Find $S_{20}$. [2 marks]

Markscheme

correct substitution (A1)
e.g.
$S_{20} = \frac{20}{2}(2 + 59).$
$S_{20} = \frac{20}{2}(2 \times 2 + 19 \times 3)$
$S_{20} = 610 \quad A1 \quad N2$

In an arithmetic sequence
$u_1 = 7,$
$u_{20} = 64$ and
$u_n = 3709.$

15a. Find the value of the common difference. [3 marks]

Markscheme

evidence of choosing the formula for 20th term (M1)
e.g.
$u_{20} = u_1 + 19d$
correct equation A1
e.g.
$64 = 7 + 19d,$
$d = \frac{64 - 7}{19}$
$d = 3 \quad A1 \quad N2$

15b. Find the value of $n$. [2 marks]
Consider the arithmetic sequence 3, 9, 15, \ldots, 1353.

16a. Write down the common difference.

**Markscheme**

common difference is 6  \( A1 \)  \( N1 \)  

**[1 mark]**

16b. Find the number of terms in the sequence.

**Markscheme**

evidence of appropriate approach  (\( M1 \))

e.g.
\( u_0 = 1353 \)
correct working  \( A1 \)

e.g.
\[
1353 = 3 + (n - 1)6, \\
\frac{1353 + 3}{6}
\]
\( n = 226 \)  \( A1 \)  \( N2 \)  

**[3 marks]**

16c. Find the sum of the sequence.

**Markscheme**

evidence of correct substitution  \( A1 \)

e.g.
\[
S_{226} = \frac{226(3+1353)}{2}, \\
= \frac{226(2 \times 3 + 225 \times 6)}{2}
\]
\( S_{226} = 153228 \) (accept 153000)  \( A1 \)  \( N1 \)  

**[2 marks]**
An arithmetic sequence, \( u_1, u_2, u_3 \ldots \), has 
\( d = 11 \) and 
\( u_{27} = 263. \)

17a. Find \( u_1. \) [2 marks]

**Markscheme**

evidence of equation for \( u_{27} \) \( M1 \)

e.g.
\[ 263 = u_1 + 26 \times 11, \]
\[ u_{27} = u_1 + (n - 1) \times 11, \]
\[ 263 = 11 \times (26) \]
\[ u_1 = -23 \quad A1 \quad N1 \]

[2 marks]

17b. (i) Given that \( u_n = 516 \), find the value of \( n \).

(ii) For this value of \( n \), find \( S_n \). [4 marks]

**Markscheme**

(i) correct equation \( A1 \)

e.g.
\[ 516 = -23 + (n - 1) \times 11, \]
\[ 539 = (n - 1) \times 11 \]
\[ n = 50 \quad A1 \quad N1 \]

(ii) correct substitution into sum formula \( A1 \)

e.g.
\[ S_{50} = \frac{50((-23+516))}{2}, \]
\[ S_{50} = \frac{50(2\times(-23)+49\times11)}{2} \]
\[ S_{50} = 12325 \quad (\text{accept } 12300) \quad A1 \quad N1 \]

[4 marks]