Core 2 Sequences Questions

5 The $n$th term of a sequence is $u_n$.

The sequence is defined by

$$u_{n+1} = pu_n + q$$

where $p$ and $q$ are constants.

The first three terms of the sequence are given by

$$u_1 = 200 \quad u_2 = 150 \quad u_3 = 120$$

(a) Show that $p = 0.6$ and find the value of $q$. \hspace{1cm} (5 marks)

(b) Find the value of $u_4$. \hspace{1cm} (1 mark)

(c) The limit of $u_n$ as $n$ tends to infinity is $L$. Write down an equation for $L$ and hence find the value of $L$. \hspace{1cm} (3 marks)

3 The first term of an arithmetic series is 1. The common difference of the series is 6.

(a) Find the tenth term of the series. \hspace{1cm} (2 marks)

(b) The sum of the first $n$ terms of the series is 7400.

(i) Show that $3n^2 - 2n - 7400 = 0$. \hspace{1cm} (3 marks)

(ii) Find the value of $n$. \hspace{1cm} (2 marks)

4 (a) The expression $(1-2x)^4$ can be written in the form

$$1 + px + qx^2 - 32x^3 + 16x^4$$

By using the binomial expansion, or otherwise, find the values of the integers $p$ and $q$. \hspace{1cm} (3 marks)

(b) Find the coefficient of $x$ in the expansion of $(2 + x)^9$. \hspace{1cm} (2 marks)

(c) Find the coefficient of $x$ in the expansion of $(1 - 2x)^4 (2 + x)^9$. \hspace{1cm} (3 marks)
5 The second term of a geometric series is 48 and the fourth term is 3.

(a) Show that one possible value for the common ratio, \( r \), of the series is \(-\frac{1}{4}\) and state the other value. (4 marks)

(b) In the case when \( r = -\frac{1}{4} \), find:

(i) the first term; (1 mark)

(ii) the sum to infinity of the series. (2 marks)

7 (a) The first four terms of the binomial expansion of \((1 + 2x)^8\) in ascending powers of \(x\) are \(1 + ax + bx^2 + cx^3\). Find the values of the integers \(a\), \(b\) and \(c\). (4 marks)

(b) Hence find the coefficient of \(x^3\) in the expansion of \((1 + \frac{1}{2}x)(1 + 2x)^8\). (3 marks)

2 The \(n\)th term of a geometric sequence is \(u_n\), where

\[ u_n = 3 \times 4^n \]

(a) Find the value of \(u_1\) and show that \(u_2 = 48\). (2 marks)

(b) Write down the common ratio of the geometric sequence. (1 mark)

(c) (i) Show that the sum of the first 12 terms of the geometric sequence is \(4^k - 4\), where \(k\) is an integer. (3 marks)

(ii) Hence find the value of \(\sum_{n=2}^{12} u_n\). (1 mark)

4 An arithmetic series has first term \(a\) and common difference \(d\).

The sum of the first 29 terms is 1102.

(a) Show that \(a + 14d = 38\). (3 marks)

(b) The sum of the second term and the seventh term is 13.

Find the value of \(a\) and the value of \(d\). (4 marks)
Core 2 Sequences Answers

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
<th>Mark</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>5(a)</td>
<td>$150 = 200p + q$ $120 = 150p + q$</td>
<td>M1</td>
<td>Either equation</td>
</tr>
<tr>
<td></td>
<td>$p = 0.6$ $q = 30$</td>
<td>A1</td>
<td>Both (condone embedded values for the M1A1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>m1</td>
<td>Valid method to solve two simultaneous eqns in $p$ and $q$ to find either $p$ or $q$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1</td>
<td>AG (condone if left as a fraction)</td>
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<tr>
<td></td>
<td></td>
<td>B1</td>
<td>5</td>
</tr>
<tr>
<td>(b)</td>
<td>$u_4 = 102$</td>
<td>B1F</td>
<td>1</td>
</tr>
<tr>
<td>(c)</td>
<td>$L = pL + q$ $L = 0.6L + 30$</td>
<td>M1</td>
<td>Ft on $(72 + q)$</td>
</tr>
<tr>
<td></td>
<td>$L = \frac{q}{1 - p}$</td>
<td>m1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$L = 75$</td>
<td>A1F</td>
<td>Ft on $2.5q$</td>
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<td>Total 9</td>
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<th>Question</th>
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<tbody>
<tr>
<td>3(a)</td>
<td>(Tenth term) $a + (10 − 1)d$ $\ldots = 1 + 9(6) = 55$</td>
<td>M1</td>
<td>NMS or rep. addn. B2 CAO</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A1</td>
<td>SC if M0 award B1 for $6n − 5$ OE</td>
</tr>
<tr>
<td>(b)(i)</td>
<td>$S_n = \frac{n}{2}[2 + (n − 1)6]$</td>
<td>M1</td>
<td>Formula for $(S_n)$ with either $a = 1$ or $d = 6$ substituted</td>
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<tr>
<td></td>
<td></td>
<td>A1</td>
<td>Eqn formed with some expansion of brackets</td>
</tr>
<tr>
<td></td>
<td>$\frac{n}{2}[2 + 6n − 6] = 7400$</td>
<td></td>
<td>CSO AG</td>
</tr>
<tr>
<td></td>
<td>$3n^2 − 2n = 7400 \Rightarrow 3n^2 − 2n − 7400 = 0$</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>$(3n + 14S)(n − 50) = 0$</td>
<td>M1</td>
<td>Formula/factorisation OE</td>
</tr>
<tr>
<td></td>
<td>$\Rightarrow n = 50$</td>
<td>A1</td>
<td>NMS single ans. 50, B2 CAO</td>
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<td>NMS 50 and −49 (3….) B1 CAO</td>
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<td>Total 7</td>
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</table>
(a) \((1-2x)^4 = (1)^4 + 4(1)^3(-2x) + 6(1)^2(-2x)^2 + 4(1)(-2x)^3 + (-2x)^4\)
\[= 1 - 8x + 24x^2 - 32x^3 + 16x^4\]
\[= [1] - 8x + 24x^2 - 32x^3 + 16x^4\]

M1: Any valid method as far as term(s) in \(x\) and term(s) in \(x^2\).

A1: \(p = -8\) Accept \(-8x\) even within a series.

A1: \(q = 24\) Accept \(24x^2\) even within a series.

(b) \(x\) term is \(\binom{9}{1}2^9x\)

M1: OE

A1: Coefficient of \(x\) term is \(9 \times 2^9 = 2304\) (=8)

A1: Condone 2304x

(c) \((1-2x)^4(2+x)^3 = (1+pxr\ldots)(2^9-kx\ldots)\)

= ....

= \ldots + kx + px(2^9) + \ldots\)

M1: Uses (a) and (b) oe (PI)

Multiply the two expansions to get \(x\) terms

M1: Coefficient of \(x\) is \(k + 512p\)

A1f: 3 ft on candidate’s values of \(k\) and \(p\).

Condone \(-1792x\)

SC If 0/3 award B1ft for \(p+k\) evaluated

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5(a) \(ar = 48, \quad ar^3 = 3\)

M1: For either, OE

B1: Elimination of \(a\) OE

\(r^3 = \frac{1}{16} \iff r = -\frac{1}{4}\)

A1: CSO AG Full valid completion.

SC Clear explicit verification (max B2 out of 3.)

\(or \quad r = \frac{1}{4}\)

B1: 4

(b)(i) \(a = -192\)

B1: 1

\(\frac{a}{1-r} = \frac{a}{\frac{1}{4}}\)

M1: \(\frac{a}{1-r}\) used

\(S_n = \frac{-768}{5} \quad (\approx -153.6)\)

A1f: 2

SC candidate uses \(r = 0.25\),

gives \(a = -192\) and

sum to infinity \(= 256\).

(max. B0 M1 A1)

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Total 8

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Total 7
7(a) \[(1 + 2x)^5\]
\[= 1 + \binom{5}{1}(2x)^1 + \binom{5}{2}(2x)^2 + \binom{5}{3}(2x)^3 + \binom{5}{4}(2x)^4 + \binom{5}{5}(2x)^5 + \ldots\]
\[= 1 + 10x + 112x^2 + 448x^3 + \ldots\]
\[(a = 16, \ b = 112, \ c = 448)\]
- A1A1
- A1 for each of \(a, b, c\)

(b) \(x^3\) terms from expansion of \((1 + \frac{1}{2}x)(1 + 2x)^5\)
- \(cx^3\) and \(\frac{1}{2}x(5x^2)\)
- A1
- Either \(b, c\) or candidate's values for \(b\) and \(c\) from (a)
- Coefficient of \(x^3\) is \(c + 0.5b = 504\)
- A1ft
- Ft on candidate's \((c + 0.5b)\) provided \(b\) and \(c\) are positive integers \(\geq 1\)

Total 7

2(a) \(u_1 = 12\)
\(u_2 = 3 \times 4^2 = 48\)
- B1
- B1
- CSO AG (be convinced)

(b) \(r = 4\)
- B1
- 1

(c)(i) \(S_n = \frac{\alpha(1-r^n)}{1-r}\)
- M1
- OE Using a correct formula with \(n = 12\)
- \(\frac{12(1-4^{12})}{1-4}\)
- A1ft
- Ft on answer for \(u_1\) in (a) and \(r\) in (b)
- \(-4(1-4^{12}) = 4^{13} - 4\)
- A1
- CAO Accept \(k = 13\) for \(4^{13}\) term

(ii) \(\sum_{n=2}^{12} u_n = (4^{13} - 4) - u_1\)
- \(= 67108848\)
- B1
- 1

Total 7
4(a) \[
\left\{S_{29}\right\} = \frac{29}{2} \left[ 2a + 28d \right]
\]

\[29(a + 14d) = 1102\]

\[a + 14d = \frac{1102}{29} \Rightarrow a + 14d = 38\]

M1

Equation formed then some manipulation

m1

A1 3

CSO AG

(b) \[\begin{align*}
\quad u_2 &= a + d \\
\quad u_7 &= a + 6d \\
\quad u_2 + u_7 &= 13 \Rightarrow 2a + 7d = 13
\end{align*}\]

\[\text{e.g. } 21d = 63; \quad 3a = -12\]

B1

Either expression correct

M1

Forming equation using \(u_2\) & \(u_7\) both in form \(a + kd\)

m1

Solving \(a + 14d = 38\) with candidate’s ‘\(2a + 7d = 13\)’ to at least stage of elimination of either \(a\) or \(d\)

A1 4

Both correct

Total 7