Paper 1 markscheme

1. [Maximum mark: 6] 

The following box-and-whisker plot shows the number of tweets sent by people in a coffee shop on a particular day.

(a) Find the value of the interquartile range. 

\[ \text{IQR} = Q_3 - Q_1 \]
\[ = 7 - 2 \]
\[ = 5 \]

(b) One person sent \( k \) tweets, where \( k > 7 \). Given that \( k \) is an outlier, find the least value of \( k \).

\[ Q_3 + 1.5 \times \text{IQR} \]
\[ = 7 + 1.5 \times 5 \]
\[ = 14.5 \]
\[ \therefore k = 15 \]

2. [Maximum mark: 6] 

Consider the following sequence of figures.

Figure 1 contains 6 line segments.

(a) Given that Figure \( n \) contains 101 line segments, show that \( n = 20 \).

(b) Find the total number of line segments in the first 20 figures.

3.
Let \( f(x) = ax^2 - 24x + c \). A horizontal line, \( L \), intersects the graph of \( f \) at \( x = 1 \) and \( x = 7 \).

(a) (i) The equation of the axis of symmetry is \( x = h \). Find \( h \).
(ii) Hence, show that \( a = 3 \).

(b) The equation of \( L \) is \( y = 6 \). Find the value of \( c \).

\[ a) \ (i) \ \text{A.O.S} \ x = \frac{b}{2a} \]
\[ \therefore 4 = \frac{-(-24)}{2a} \]
\[ \implies 2a = \frac{24}{4} \implies a = 3 \]
\[ b) f(x) = 3x^2 - 24x + c \]
\[ 6 = 3(1)^2 - 24(1) + c \]
\[ 6 = 3 - 24 + c \]
\[ c = 27 \]

4. [Maximum mark: 7]

The following diagram shows an archery target which is divided into three regions A, B and C.

A contest consists of an archer shooting one arrow at the target. The probability of hitting each region is given in the following table.

<table>
<thead>
<tr>
<th>Region</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>not hit</th>
<th>\text{Total}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>$\frac{1}{24}$</td>
<td>$\frac{4}{24}$</td>
<td>$\frac{7}{24}$</td>
<td>$\frac{1}{2}$</td>
<td>1</td>
</tr>
</tbody>
</table>

(a) Find the probability that the arrow does not hit the target.

The archer scores points as shown in the following table.

<table>
<thead>
<tr>
<th>Region</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Outside Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Points</td>
<td>10</td>
<td>6</td>
<td>( k )</td>
<td>-4</td>
</tr>
</tbody>
</table>

(b) Given that the contest is fair, find the value of \( k \).
Five equilateral triangles, each with side length 4 cm, are arranged to form a truss bridge model.

This is shown in the following diagram.

The vectors \( \mathbf{a}, \mathbf{b} \) and \( \mathbf{c} \) are shown on the diagram.

Find \( \mathbf{a} \cdot (\mathbf{a} + \mathbf{b} + 2\mathbf{c}) \).

\[
\mathbf{a} \cdot 3\mathbf{c} = |\mathbf{a}| |3\mathbf{c}| \cos \theta
\]

\[
\mathbf{a} \cdot 3\mathbf{c} = 4 \times 12 \times \frac{1}{2}
\]

\[
\mathbf{a} \cdot 3\mathbf{c} = 24
\]

6.

[Maximum mark: 7]

The expression \( 8\sin x \cos x \) can be written in the form \( p\sin qx \).

(a) Find the value of \( p \) and the value of \( q \).

(b) Hence or otherwise, solve the equation \( 8\sin x \cos x = 2\sqrt{3} \), for \( \frac{\pi}{4} \leq x \leq \frac{\pi}{2} \).

\[
\frac{\pi}{4} \quad 90^\circ
\]

\[
\begin{align*}
\text{a) using} & \quad \sin 2x = 2\sin x \cos x \\
4\sin 2x & = 8\sin x \cos x \\
p = 4 & \quad q = 2
\end{align*}
\]

\[
\begin{align*}
\text{b) } 4\sin 2x & = 2\sqrt{3} \\
\sin 2x & = \frac{\sqrt{3}}{2} \\
2x & = \frac{\pi}{3}, \frac{2\pi}{3} \\
x & = \frac{\pi}{6}, \frac{2\pi}{3}
\end{align*}
\]

\( \text{not in domain} \)

7.
8. [Maximum mark: 7]

Let \( f(x) = 6x - \ln x \), for \( x > 0 \).

(a) Find \( f'(x) \).
(b) Find \( f''(x) \).
(c) Solve \( f'(x) = f''(x) \).

\[
\begin{align*}
a &= 6 \\
b &= -1 \\
c &= -1 \\
\frac{a}{b} &= 6 \\
\frac{c}{b} &= -1 \\
(6x - 3) &= 3 \\
(2x - 1) &= 1 \\
(6x + 2) &= 2 \\
(3x + 1) &= 1 \\
&= 1
\end{align*}
\]

\[
\begin{align*}
a &= 6 \\
b &= -1 \\
c &= -1 \\
\frac{a}{b} &= 6 \\
\frac{c}{b} &= -1 \\
(6x - 3) &= 3 \\
(2x - 1) &= 1 \\
(6x + 2) &= 2 \\
(3x + 1) &= 1 \\
&= 1
\end{align*}
\]

8. [Maximum mark: 13]

A function \( f(x) \) has derivative \( f'(x) = 6x^2 - 24x \). The graph of \( f \) has an \( x \)-intercept at \( x = 1 \).

(a) Find \( f(x) \).
(b) The graph of \( f \) has a point of inflexion at \( x = k \). Find \( k \).
(c) Find the values of \( x \) for which the graph of \( f \) is concave-up.

\[
\begin{align*}
\int f'(x) \, dx &= \int (6x^2 - 24x) \, dx \\
&= 2x^3 - 12x^2 + C \\
f(x) &= 2x^3 - 12x^2 + C \\
o &= 2(1)^3 - 12(1)^2 + C \\
C &= 10 \\
\therefore f(x) &= 2x^3 - 12x^2 + 10
\end{align*}
\]

9.
10. [Maximum mark: 15] [X]

The first two terms of an infinite geometric sequence are \( u_1 = 20 \) and \( u_2 = 16 \sin^2 \theta \), where \( 0 < \theta < 2\pi \), and \( \theta \neq \pi \).

(a) (i) Find an expression for \( r \) in terms of \( \theta \).

(ii) Find the possible values of \( r \).

(b) Show that the sum of the infinite sequence is \( \frac{100}{3 + 2 \cos 2\theta} \).

(c) Find the values of \( \theta \) which give the greatest value of the sum.

\[
\cos 2\theta = 1 - 2 \sin^2 \theta \\
2 \cos 2\theta = 2 - 4 \sin^2 \theta \\
2 \cos 2\theta + 3 = 5 - 4 \sin^2 \theta
\]

\[
\frac{100}{3 + 2 (-1)} = \frac{100}{1} = 100
\]

\[
\theta = \pi, \frac{3\pi}{2}, \frac{5\pi}{2}
\]

10. [Maximum mark: 15] [X]

A closed cylindrical can with radius \( r \) cm and height \( h \) cm has a volume of 24\( \pi \) cm\(^3\).

\( V = \pi r^2 h \)

\( 24\pi = \pi r^2 h \)

\( h = \frac{24}{r^2} \)

(a) Express \( h \) in terms of \( r \).

The material for the base and top of the can costs 15 cents per cm\(^2\) and the material for the curved side costs 10 cents per cm\(^2\). The total cost of the material, in cents, is \( C \).

(b) Show that \( C = 30\pi r^2 + \frac{1480\pi}{r} \).

\( C = 30\pi r^2 + 20\pi r h \)

\( C = 30\pi r^2 + 20\pi r \left( \frac{24}{r^2} \right) \)

\( C = 30\pi r^2 + \frac{480\pi}{r} \)

\( C = 30\pi r^2 + 480\pi r^{-1} \)

\( C = 60\pi r - \frac{480\pi}{r^2} \)

\( 8 = \frac{r^3}{2} \)

\( r = 2 \)

\( C = 30\pi (2)^2 + \frac{480\pi}{2} \)

\( C = 120\pi + 240\pi \)

\( C = 360\pi \)