Purpose of internal assessment

Internal assessment is an integral part of the course and is compulsory for both SL and HL students. It enables students to demonstrate the application of their skills and knowledge and to pursue their personal interests without the time limitations and other constraints that are associated with written examinations. The internal assessment should, as far as possible, be woven into normal classroom teaching and not be a separate activity conducted after a course has been taught.

The internal assessment requirements at SL and at HL is an individual exploration. This is a piece of written work that involves investigating an area of mathematics. It is marked according to five assessment criteria.

Guidance and authenticity

The exploration submitted for internal assessment must be the student’s own work. However, it is not the intention that students should decide upon a title or topic and be left to work on the internal assessment component without any further support from the teacher. The teacher should play an important role during both the planning stage and the period when the student is working on the exploration.

It is the responsibility of the teacher to ensure that students are familiar with:

- the requirements of the type of work to be internally assessed
- the IB academic honesty policy available on the programme resource centre
- the assessment criteria; students must understand that the work submitted for assessment must address these criteria effectively.

Teachers and students must discuss the exploration. Students should be encouraged to initiate discussions with the teacher to obtain advice and information, and students must not be penalized for seeking guidance. As part of the learning process, teachers should read and give advice to students on one draft of the work. The teacher should provide oral or written advice on how the work could be improved, but not edit the draft. The next version handed to the teacher must be the final version for submission.

It is the responsibility of teachers to ensure that all students understand the basic meaning and significance of concepts that relate to academic honesty, especially authenticity and intellectual property. Teachers must ensure that all student work for assessment is prepared according to requirements and must explain clearly to students that the internally assessed work must be entirely their own.

All work submitted to the IB for moderation or assessment must be authenticated by a teacher, and must not include any known instances of suspected or confirmed malpractice. Each student must confirm that the work is his or her authentic work and constitutes the final version of that work. Once a student has officially submitted the final version of the work it cannot be retracted. The requirement to confirm the authenticity of work applies to the work of all students, not just the sample work that will be submitted to the IB for the purpose of moderation. For further details refer to the IB publication Academic honesty in the IB educational context, The Diploma Programme: From principles into practice and the relevant articles in General regulations: Diploma Programme.

Authenticity may be checked by discussion with the student on the content of the work, and scrutiny of one or more of the following:

- the student's initial proposal
- the draft of the written work
- the references cited
- the style of writing compared with work known to be that of the student
• the analysis of the work by a web-based plagiarism detection service such as www.turnitin.com.
The same piece of work cannot be submitted to meet the requirements of both the internal assessment and
the extended essay.

Collaboration and teamwork

Collaboration and teamwork are a key focus of the approaches to teaching in the DP. It is advisable that the
teacher uses the available class time to manage student collaboration. While working on their exploration
students should be encouraged to work collaboratively in the various phases of the process, for example:
• generating ideas
• selecting the topic for their exploration
• sharing research sources
• acquiring the necessary knowledge, skills and understanding
• seeking peer feedback on their writing.

The Approaches to teaching and learning (ATL) website on the programme resource centre provides an
excellent source for developing collaborative skills in students.

While students should be encouraged to talk through their ideas with others, it is not appropriate to work
together on a single exploration. It is important that students demonstrate how they incorporated sources
and collaborative ideas into their work and that they always show their understanding and engagement in
the work as described in the assessment criteria. Marks are awarded for the student’s development and
contribution to their exploration, not for work found in literature or carried out by others either individually
or collaboratively.

It is imperative that students understand that the writing and calculations they do in their work must
always be their own. This means that the argument they make and the ideas they rely on to make it, should
either be their own or they should give credit to the source of those ideas. Any sources must be cited
accordingly. This includes pictures, diagrams, graphs, formulae, etc.

In the specific cases of collecting information, data or measurements it is imperative that each student
collects their own data, even in a case where collection of measurements is made from a group experiment.
Group data or measurements can be combined to provide enough information for individual analysis and
this should be clearly described in the written exploration.

Time allocation

Internal assessment is an integral part of the mathematics courses, contributing 20% to the final assessment
in the SL and the HL courses. This weighting should be reflected in the time that is allocated to teaching the
knowledge, skills and understanding required to undertake the work, as well as the total time allocated to
carry out the work.

It is recommended that a total of approximately 10-15 hours of teaching time should be allocated to the
work. This should include:
• time for the teacher to explain to students the requirements of the exploration
• class time for students to work on the exploration and ask questions
• time for consultation between the teacher and each student
• time to review and monitor progress, and to check authenticity.

Requirements and recommendations

Students can choose from a wide variety of activities: for example, modelling, investigations and
applications of mathematics. To assist teachers and students in the choice of a topic, a list of stimuli is
available in the teacher support material. However, students are not restricted to this list.
The exploration should be approximately 12-20 pages long with double line spacing, including diagrams and graphs, but excluding the bibliography. However, it is the quality of the mathematical writing that is important, not the length.

The teacher is expected to give appropriate guidance at all stages of the exploration by, for example, directing students into more productive routes of inquiry, making suggestions for suitable sources of information, and providing advice on the content and clarity of the exploration in the writing-up stage.

Teachers are responsible for indicating to students the existence of errors but should not explicitly correct these errors. It must be emphasized that students are expected to consult the teacher throughout the process.

All students should be familiar with the requirements of the exploration and the criteria by which it is assessed. Students need to start planning their explorations as early as possible in the course. Deadlines should be firmly established and adhered to. There should be a date for submission of the exploration topic and a brief outline description, a date for the submission of the draft and, of course, a date for completion.

In developing their explorations, students should aim to make use of mathematics learned as part of the course. The mathematics used should be commensurate with the level of the course—that is, it should be similar to that suggested in the syllabus. It is not expected that students produce work that is outside the syllabus—however, this will not be penalized.

Ethical guidelines should be adhered to throughout the planning and conducting of the exploration. Further details are given in the Ethical practice in the Diploma Programme poster on the programme resource centre.

Presentation

The following details should be stated on the cover page of the exploration:

- title of the exploration
- number of pages.

The references are not assessed. However, if they are not included in the final report it may be flagged in terms of academic honesty.

Using assessment criteria for internal assessment

For internal assessment, a number of assessment criteria have been identified. Each assessment criterion has level descriptors describing specific achievement levels, together with an appropriate range of marks. The level descriptors concentrate on positive achievement, although for the lower levels failure to achieve may be included in the description.

Teachers must judge the internally assessed work at SL and at HL against the criteria using the level descriptors.

The assessment criteria A to D are the same for both SL and HL. Criterion E “Use of mathematics” is different for SL and HL.

The aim is to find, for each criterion, the descriptor that conveys most accurately the level attained by the student, using the best-fit model. A best-fit approach means that compensation should be made when a piece of work matches different aspects of a criterion at different levels. The mark awarded should be one that most fairly reflects the balance of achievement against the criterion. It is not necessary for every single aspect of a level descriptor to be met for that mark to be awarded.

When assessing a student’s work, teachers should read the level descriptors for each criterion until they reach a descriptor that most appropriately describes the level of the work being assessed. If a piece of work seems to fall between two descriptors, both descriptors should be read again and the one that more appropriately describes the student’s work should be chosen.

Where there are two or more marks available within a level, teachers should award the upper marks if the student’s work demonstrates the qualities described to a great extent; the work may be close to achieving...
marks in the level above. Teachers should award the lower marks if the student’s work demonstrates the qualities described to a lesser extent; the work may be close to achieving marks in the level below.

Only whole numbers should be recorded; partial marks, (fractions or decimals) are not acceptable.

Teachers should not think in terms of a pass or fail boundary, but should concentrate on identifying the appropriate descriptor for each assessment criterion.

The highest-level descriptors do not imply faultless performance but should be achievable by a student. Teachers should not hesitate to use the extremes if they are appropriate descriptions of the work being assessed.

A student who attains a high achievement level in relation to one criterion will not necessarily attain high achievement levels in relation to the other criteria. Similarly, a student who attains a low achievement level for one criterion will not necessarily attain low achievement levels for the other criteria. Teachers should not assume that the overall assessment of the students will produce any particular distribution of marks.

It is recommended that the assessment criteria be made available to students.

Internal assessment details

Mathematical exploration

Duration: 10 to 15 hours

Weighting: 20%

Introduction

The internally-assessed component in this course is a mathematical exploration. This is a short report written by the student based on a topic chosen by him or her, and it should focus on the mathematics of that particular area. The emphasis is on mathematical communication (including formulae, diagrams, graphs, tables and so on), with his or her own focus, with the teacher providing feedback via, for example, discussion and interview. This will allow the students to develop areas of interest to them without a time constraint as in an examination, and allow all students to experience a feeling of success.

The final report should be approximately 12-20 pages long with double line spacing. It can be either word processed or handwritten. Students should be able to explain all stages of their work in such a way that demonstrates clear understanding. While there is no requirement that students present their work in class, it should be written in such a way that their peers would be able to follow it fairly easily. The report should include a detailed bibliography, and sources need to be referenced in line with the IB academic honesty policy. Direct quotes must be acknowledged.

The purpose of the exploration

The aims of the Mathematics: analysis and approaches and Mathematics: applications and interpretation courses at both SL and HL are carried through into the objectives that are formally assessed as part of the course, through either written examination papers or the exploration, or both. In addition to testing the objectives of the course, the exploration is intended to provide students with opportunities to increase their understanding of mathematical concepts and processes, and to develop a wider appreciation of mathematics. These are noted in the aims of the course. It is intended that, by doing the exploration, students benefit from the mathematical activities undertaken and find them both stimulating and rewarding. It will enable students to acquire the attributes of the IB learner profile.

The specific purposes of the exploration are to:

- develop students’ personal insight into the nature of mathematics and to develop their ability to ask their own questions about mathematics
- provide opportunities for students to complete a piece of mathematical work over an extended period of time
- enable students to experience the satisfaction of applying mathematical processes independently
• provide students with the opportunity to experience for themselves the beauty, power and usefulness of mathematics
• encourage students, where appropriate, to discover, use and appreciate the power of technology as a mathematical tool
• enable students to develop the qualities of patience and persistence, and to reflect on the significance of their work
• provide opportunities for students to show, with confidence, how they have developed mathematically.

Management of the exploration

Work on the exploration should be incorporated into the course so that students are given the opportunity to learn the skills needed. Time in class can therefore be used for general discussion of areas of study, as well as familiarizing students with the criteria. Further details on the development of the exploration are included in the teacher support material.

Internal assessment criteria—SL and HL

The exploration is internally assessed by the teacher and externally moderated by the IB using assessment criteria that relate to the objectives for mathematics. Each exploration is assessed against the following five criteria. The final mark for each exploration is the sum of the scores for each criterion. The maximum possible final mark is 20.

Students will not receive a grade for their mathematics course if they have not submitted an exploration.

<table>
<thead>
<tr>
<th>Criterion A</th>
<th>Presentation</th>
</tr>
</thead>
<tbody>
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<td>Criterion B</td>
<td>Mathematical communication</td>
</tr>
<tr>
<td>Criterion C</td>
<td>Personal engagement</td>
</tr>
<tr>
<td>Criterion D</td>
<td>Reflection</td>
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<tr>
<td>Criterion E</td>
<td>Use of mathematics</td>
</tr>
</tbody>
</table>

Criterion A: Presentation

<table>
<thead>
<tr>
<th>Achievement level</th>
<th>Descriptor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>The exploration does not reach the standard described by the descriptors below.</td>
</tr>
<tr>
<td>1</td>
<td>The exploration has some coherence or some organization.</td>
</tr>
<tr>
<td>2</td>
<td>The exploration has some coherence and shows some organization.</td>
</tr>
<tr>
<td>3</td>
<td>The exploration is coherent and well organized.</td>
</tr>
<tr>
<td>4</td>
<td>The exploration is coherent, well organized, and concise.</td>
</tr>
</tbody>
</table>

The “presentation” criterion assesses the organization and coherence of the exploration.

A coherent exploration is logically developed, easy to follow and meets its aim. This refers to the overall structure or framework, including introduction, body, conclusion and how well the different parts link to each other.

A well-organized exploration includes an introduction, describes the aim of the exploration and has a conclusion. Relevant graphs, tables and diagrams should accompany the work in the appropriate place and not be attached as appendices to the document. Appendices should be used to include information on large data sets, additional graphs, diagrams and tables.
A **concise** exploration does not show irrelevant or unnecessary repetitive calculations, graphs or descriptions.

The use of technology is not required but encouraged where appropriate. However, the use of analytic approaches rather than technological ones does not necessarily mean lack of conciseness, and should not be penalized. This does not mean that repetitive calculations are condoned.

**Criterion B: Mathematical communication**

<table>
<thead>
<tr>
<th>Achievement level</th>
<th>Descriptor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>The exploration does not reach the standard described by the descriptors below.</td>
</tr>
<tr>
<td>1</td>
<td>The exploration contains some relevant mathematical communication which is partially appropriate.</td>
</tr>
<tr>
<td>2</td>
<td>The exploration contains some relevant appropriate mathematical communication.</td>
</tr>
<tr>
<td>3</td>
<td>The mathematical communication is relevant, appropriate and is mostly consistent.</td>
</tr>
<tr>
<td>4</td>
<td>The mathematical communication is relevant, appropriate and consistent throughout.</td>
</tr>
</tbody>
</table>

The “mathematical communication” criterion assesses to what extent the student has:

- used appropriate mathematical language (notation, symbols, terminology). Calculator and computer notation is acceptable only if it is software generated. Otherwise it is expected that students use appropriate mathematical notation in their work
- defined key terms and variables, where required
- used multiple forms of mathematical representation, such as formulae, diagrams, tables, charts, graphs and models, where appropriate
- used a deductive method and set out proofs logically where appropriate

Examples of level 1 can include graphs not being labelled, consistent use of computer notation with no other forms of correct mathematical communication.

Level 4 can be achieved by using only one form of mathematical representation as long as this is appropriate to the topic being explored. For level 4, any minor errors that do not impair clear communication should not be penalized.

**Criterion C: Personal engagement**

<table>
<thead>
<tr>
<th>Achievement level</th>
<th>Descriptor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>The exploration does not reach the standard described by the descriptors below.</td>
</tr>
<tr>
<td>1</td>
<td>There is evidence of some personal engagement.</td>
</tr>
<tr>
<td>2</td>
<td>There is evidence of significant personal engagement.</td>
</tr>
<tr>
<td>3</td>
<td>There is evidence of outstanding personal engagement.</td>
</tr>
</tbody>
</table>

The “personal engagement” criterion assesses the extent to which the student engages with the topic by exploring the mathematics and making it their own. It is not a measure of effort.

Personal engagement may be recognized in different ways. These include thinking independently or creatively, presenting mathematical ideas in their own way, exploring the topic from different perspectives, making and testing predictions. Further (but not exhaustive) examples of personal engagement at different levels are given in the teacher support material (TSM).
There must be evidence of personal engagement demonstrated in the student’s work. It is not sufficient that a teacher comments that a student was highly engaged.

Textbook style explorations or reproduction of readily available mathematics without the candidate’s own perspective are unlikely to achieve the higher levels.

**Significant:** The student demonstrates authentic personal engagement in the exploration on a few occasions and it is evident that these drive the exploration forward and help the reader to better understand the writer’s intentions.

**Outstanding:** The student demonstrates authentic personal engagement in the exploration in numerous instances and they are of a high quality. It is evident that these drive the exploration forward in a creative way. It leaves the impression that the student has developed, through their approach, a complete understanding of the context of the exploration topic and the reader better understands the writer’s intentions.

**Criterion D: Reflection**

<table>
<thead>
<tr>
<th>Achievement level</th>
<th>Descriptor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>The exploration does not reach the standard described by the descriptors below.</td>
</tr>
<tr>
<td>1</td>
<td>There is evidence of limited reflection.</td>
</tr>
<tr>
<td>2</td>
<td>There is evidence of meaningful reflection.</td>
</tr>
<tr>
<td>3</td>
<td>There is substantial evidence of critical reflection.</td>
</tr>
</tbody>
</table>

The “reflection” criterion assesses how the student reviews, analyses and evaluates the exploration. Although reflection may be seen in the conclusion to the exploration, it may also be found throughout the exploration.

Simply describing results represents **limited reflection**. Further consideration is required to achieve the higher levels.

Some ways of showing **meaningful reflection** are: linking to the aims of the exploration, commenting on what they have learned, considering some limitation or comparing different mathematical approaches.

**Critical reflection** is reflection that is crucial, deciding or deeply insightful. It will often develop the exploration by addressing the mathematical results and their impact on the student’s understanding of the topic. Some ways of showing critical reflection are: considering what next, discussing implications of results, discussing strengths and weaknesses of approaches, and considering different perspectives.

**Substantial evidence** means that the critical reflection is present throughout the exploration. If it appears at the end of the exploration it must be of high quality and demonstrate how it developed the exploration in order to achieve a level 3.

Further (but not exhaustive) examples of reflection at different levels are given in the teacher support material (TSM).

**Criterion E: Use of mathematics—SL**

<table>
<thead>
<tr>
<th>Achievement level</th>
<th>Descriptor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>The exploration does not reach the standard described by the descriptors below.</td>
</tr>
<tr>
<td>1</td>
<td>Some relevant mathematics is used.</td>
</tr>
<tr>
<td>2</td>
<td>Some relevant mathematics is used. Limited understanding is demonstrated.</td>
</tr>
<tr>
<td>3</td>
<td>Relevant mathematics commensurate with the level of the course is used. Limited understanding is demonstrated.</td>
</tr>
</tbody>
</table>
Achievement level | Descriptor
---|---
4 | Relevant mathematics commensurate with the level of the course is used. The mathematics explored is partially correct. Some knowledge and understanding are demonstrated.
5 | Relevant mathematics commensurate with the level of the course is used. The mathematics explored is mostly correct. Good knowledge and understanding are demonstrated.
6 | Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Thorough knowledge and understanding are demonstrated.

The “Use of mathematics” SL criterion assesses to what extent students use mathematics that is relevant to the exploration.

**Relevant** refers to mathematics that supports the development of the exploration towards the completion of its aim. Overly complicated mathematics where simple mathematics would suffice is not relevant.

Students are expected to produce work that is commensurate with the level of the course, which means it should not be completely based on mathematics listed in the prior learning. The mathematics explored should either be part of the syllabus, or at a similar level.

A key word in the descriptor is **demonstrated**. The command term demonstrate means “to make clear by reasoning or evidence, illustrating with examples or practical application”. Obtaining the correct answer is not sufficient to demonstrate understanding (even some understanding) in order to achieve level 2 or higher.

For knowledge and understanding to be **thorough** it must be demonstrated throughout.

The mathematics can be regarded as **correct** even if there are occasional minor errors as long as they do not detract from the flow of the mathematics or lead to an unreasonable outcome.

Students are encouraged to use technology to obtain results where appropriate, but **understanding must be demonstrated** in order for the student to achieve higher than level 1, for example merely substituting values into a formula does not necessarily demonstrate understanding of the results.

The mathematics only needs to be what is required to support the development of the exploration. This could be a few small elements of mathematics or even a single topic (or sub-topic) from the syllabus. It is better to do a few things well than a lot of things not so well. If the mathematics used is relevant to the topic being explored, commensurate with the level of the course and understood by the student, then it can achieve a high level in this criterion.

**Criterion E: Use of mathematics—HL**

<table>
<thead>
<tr>
<th>Achievement level</th>
<th>Descriptor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>The exploration does not reach the standard described by the descriptors below.</td>
</tr>
<tr>
<td>1</td>
<td>Some relevant mathematics is used. Limited understanding is demonstrated.</td>
</tr>
<tr>
<td>2</td>
<td>Some relevant mathematics is used. The mathematics explored is partially correct. Some knowledge and understanding is demonstrated.</td>
</tr>
<tr>
<td>3</td>
<td>Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Some knowledge and understanding are demonstrated.</td>
</tr>
</tbody>
</table>
The “Use of mathematics” HL criterion assesses to what extent students use relevant mathematics in the exploration.

Students are expected to produce work that is commensurate with the level of the course, which means it should not be completely based on mathematics listed in the prior learning. The mathematics explored should either be part of the syllabus, at a similar level or slightly beyond. However, mathematics of a level slightly beyond the syllabus is not required to achieve the highest levels.

A key word in the descriptor is demonstrated. The command term demonstrate means to make clear by reasoning or evidence, illustrating with examples or practical application. Obtaining the correct answer is not sufficient to demonstrate understanding (even some understanding) in order to achieve level 2 or higher.

For knowledge and understanding to be thorough it must be demonstrated throughout. Lines of reasoning must be shown to justify steps in the mathematical development of the exploration.

Relevant refers to mathematics that supports the development of the exploration towards the completion of its aim. Overly complicated mathematics where simple mathematics would suffice is not relevant.

The mathematics can be regarded as correct even if there are occasional minor errors as long as they do not detract from the flow of the mathematics or lead to an unreasonable outcome. Precise mathematics is error-free and uses an appropriate level of accuracy at all times.

Sophistication: To be considered as sophisticated the mathematics used should be commensurate with the HL syllabus or, if contained in the SL syllabus, the mathematics has been used in a complex way that is beyond what could reasonably be expected of an SL student. Sophistication in mathematics may include understanding and using challenging mathematical concepts, looking at a problem from different perspectives and seeing underlying structures to link different areas of mathematics.

Rigour involves clarity of logic and language when making mathematical arguments and calculations. Mathematical claims relevant to the development of the exploration must be justified or proven.

Students are encouraged to use technology to obtain results where appropriate, but understanding must be demonstrated in order for the student to achieve level 1 or higher, for example merely substituting values into a formula does not necessarily demonstrate understanding of the results.

The mathematics only needs to be what is required to support the development of the exploration. This could be a few small elements of mathematics or even a single topic (or sub-topic) from the syllabus. It is better to do a few things well than a lot of things not so well. If the mathematics used is relevant to the topic being explored, commensurate with the level of the course and understood by the student, then it can achieve a high level in this criterion.