Revision

Chapter 7 and 8
Wednesday test

- Geometric and/or arithmetic sequences/series
- Financial application (make sure you know how to use the GDC)
- Exponential function or logistic function
- Use of exponent and log laws

- Finding $a,b,c,d$ for both sine and cosine functions
- Converting between complex numbers
- Plotting complex numbers in the Argand plane
- Movement in the Argand plane
- Using complex numbers to add sinusoidal functions (either sine or cosine)

- Solving simultaneous equations using GDC up to three variables

NOTE! Trickier geometry problems will be saved for later.
Simultaneous equations

• Solving simultaneous equations using GDC up to three variables

Solve for \( x \), \( b \), and \( Z \)

\[
57.655 = 2 \log x + 3e^b - 2 \log_2 Z
\]

\[
121.814 = -\log x + 6e^b + \log_2 Z
\]
Exponential equations

The number of bacteria in two colonies, A and B, starts increasing at the same time.

The number of bacteria in colony A after $t$ hours is modelled by the function $A(t) = 12e^{0.4t}$.

a. Find the number of bacteria in colony A after initial $t$ hours.

b. Find the number of bacteria in colony A after four hours.

c. How long does it take for the number of bacteria in colony A to reach 400?

d. The number of bacteria in colony B after $t$ hours is modelled by the function $B(t) = 24e^{kt}$.

After four hours, there are 60 bacteria in colony B. Find the value of $k$.

e. The number of bacteria in colony B after $t$ hours is modelled by the function $B(t) = 24e^{kt}$.

The number of bacteria in colony A first exceeds the number of bacteria in colony B after $n$ hours, where $n \in \mathbb{Z}$. Find the value of $n$.

a. correct substitution into formula \( (A1) \)

\[
\text{eg} \quad 12e^{0.4(0)}
\]

12 bacteria in the dish \( A1 \quad N2 \)

[2 marks]

b. correct substitution into formula \( (A1) \)

\[
\text{eg} \quad 12e^{0.4(4)}
\]

59.4363 \( (A1) \)

59 bacteria in the dish (integer answer only) \( A1 \quad N3 \)

c. correct equation \( (A1) \)

\[
\text{eg} \quad A(t) = 400, \quad 12e^{0.4t} = 400
\]

valid attempt to solve \( (M1) \)

\[
\text{eg} \quad \text{graph, use of logs}
\]

8.76639

8.77 (hours) \( A1 \quad N3 \)
The number of bacteria in two colonies, A and B, starts increasing at the same time.

The number of bacteria in colony A after $t$ hours is modelled by the function $A(t) = 12e^{0.4t}$.

a. Find the number of bacteria in colony A after 1 hour.

b. Find the number of bacteria in colony A after four hours.

c. How long does it take for the number of bacteria in colony A to reach 400?

d. The number of bacteria in colony B after $t$ hours is modelled by the function $B(t) = 24e^{kt}$.

After four hours, there are 60 bacteria in colony B. Find the value of $k$.

e. The number of bacteria in colony B after $t$ hours is modelled by the function $B(t) = 24e^{kt}$.

The number of bacteria in colony A first exceeds the number of bacteria in colony B after $n$ hours, where $n \in \mathbb{Z}$. Find the value of $n$.

d. valid attempt to solve \( (M1) \)

\[ eg \quad n(4) = 60, \quad 60 = 24e^{4k} \text{, use of logs} \]

correct working \( (A1) \)

\[ eg \quad \text{sketch of intersection, } 4k = \ln 2.5 \]

\[ k = 0.229072 \]

\[ k = \frac{\ln 2.5}{4} \text{ (exact), } k = 0.229 \quad A1 \quad N3 \]
The number of bacteria in two colonies, A and B, starts increasing at the same time.

The number of bacteria in colony A after $t$ hours is modelled by the function $A(t) = 12e^{0.4t}$.

a. Find the number of bacteria in colony A at $t = 0$ hours.

b. Find the number of bacteria in colony A after four hours.

c. How long does it take for the number of bacteria in colony A to reach 400?

d. The number of bacteria in colony B after $t$ hours is modelled by the function $B(t) = 24e^{kt}$.

After four hours, there are 60 bacteria in colony B. Find the value of $k$.

e. The number of bacteria in colony B after $t$ hours is modelled by the function $B(t) = 24e^{kt}$.

The number of bacteria in colony A first exceeds the number of bacteria in colony B after $n$ hours, where $n \in \mathbb{Z}$. Find the value of $n$.

**METHOD 1**

setting up an equation or inequality (accept any variable for $n$) \((M1)\)

eg $A(t) > B(t)$, $12e^{0.4n} = 24e^{0.229n}$, $e^{0.4n} = 2e^{0.229n}$

correct working \((A1)\)

eg sketch of intersection, $e^{0.171n} = 2$

4.05521 (accept 4.05349) \((A1)\)

$n = 5$ (integer answer only) \(A1 \quad N3\)

**METHOD 2**

$A(4) = 59$, $B(4) = 60$ \(\text{from earlier work}\)

$A(5) = 88.668$, $B(5) = 75.446$ \(A1A1\)

valid reasoning \((R1)\)

eg $A(4) < B(4)$ and $A(5) > B(5)$

$n = 5$ (integer answer only) \(A1 \quad N3\)
11 P2: The following shows a portion of the graph of \( y = p + q \cos(rx) \) (where \( x \) is given in degrees).

![Graph of y = p + q \cos(rx) with a range from 0 to 300 on the x-axis and 0 to 5 on the y-axis.]

\( a \) Determine the values of the constants \( p, q \) and \( r \). (6 marks)

\( b \) Hence, using technology, solve the inequality \( p + q \cos(rx) < q + p \sin(rx) \) for \( 0^\circ \leq x \leq 180^\circ \). (4 marks)

11 \( a \) \( p = 3.5, \ q = 2, \ r = 3 \)

\( b \) \( 17.2^\circ < x < 62.6^\circ \) and \( 137.2^\circ < x \leq 180^\circ \)
Yejin plans to retire at age 60. She wants to create an annuity fund, which will pay her a monthly allowance of $4000 during her retirement. She wants to save enough money so that the payments last for 30 years. A financial advisor has told her that she can expect to earn 5% interest on her funds, compounded annually.

a. Calculate the amount Yejin needs to have saved into her annuity fund, in order to meet her retirement goal. [3 marks]

b. Yejin has just turned 28 years old. She currently has no retirement savings. She wants to save part of her salary each month into her annuity fund. [3 marks]

Calculate the amount Yejin needs to save each month, to meet her retirement goal.

a. Use of finance solver \[ M1 \]

\[ N = 360, \ I = 5\%, \ Pmt = 4000, \ FV = 0, \ PpY = 12, \ CpY = 1 \] \[ A1 \]

$755000 \text{ (correct to 3 s.f.)} \] \[ A1 \]

[3 marks]

b. \[ N = 384, \ I = 5\%, \ PV = 0, \ FV = 754638, \ PpY = 12, \ CpY = 1 \] \[ M1A1 \]

$817 \text{ per month \ (correct to 3 s.f.)} \] \[ A1 \]

[3 marks]
Sophie is planning to buy a house. She needs to take out a mortgage for $120000. She is considering two possible options.

Option 1: Repay the mortgage over 20 years, at an annual interest rate of 5%, compounded annually.

Option 2: Pay $1000 every month, at an annual interest rate of 6%, compounded annually, until the loan is fully repaid.

a.i. Calculate the monthly repayment using option 1.

a.ii. Calculate the total amount Sophie would pay, using option 1.

b.i. Calculate the number of months it will take to repay the mortgage using option 2.

b.ii. Calculate the total amount Sophie would pay, using option 2.

\[
\text{a.i. evidence of using Finance solver on GDC} \quad M1
\]

\[
\text{Monthly payment } = $785 \quad (784.60) \quad A1
\]

\[2 \text{ marks}\]

\[
\text{a.ii.} 240 \times 785 = $188000 \quad M1A1
\]

\[2 \text{ marks}\]

\[
\text{b.i.} \; N = 180.7 \quad M1A1
\]

\[3 \text{ marks}\]

\[
\text{It will take 181 months} \quad A1
\]

\[3 \text{ marks}\]

\[
\text{b.ii.} 181 \times 1000 = $181000 \quad M1A1
\]

\[2 \text{ marks}\]
Give a reason why Sophie might choose

c.i. option 1.

c.ii. option 2.

c.i. The monthly repayment is lower, she might not be able to afford $1000 per month. \( R1 \)

\[ 1 \text{ mark} \]

c.ii. the total amount to repay is lower. \( R1 \)

\[ 1 \text{ mark} \]

Sophie decides to choose option 1. At the end of 10 years, the interest rate is changed to 7%, compounded annually.

d.i. Use your answer to part (a)(i) to calculate the amount remaining on her mortgage after the first 10 years.

d.ii. Hence calculate her monthly repayment for the final 10 years.

\[ \text{d.i.}$74400 \text{ (accept }$74300) \quad M1A1 \]

\[ 2 \text{ marks} \]

d.ii. Use of finance solver with \( N = 120, PV = $74400, I = 7\% \quad A1 \]

\$855 \text{ (accept }$854 – $856) \quad A1
The first two terms of an infinite geometric sequence, in order, are

\[ 2\log_2 x, \log_2 x, \text{ where } x > 0. \]

The first three terms of an arithmetic sequence, in order, are

\[ \log_2 x, \log_2 \left( \frac{x}{2} \right), \log_2 \left( \frac{x}{4} \right), \text{ where } x > 0. \]

Let \( S_{12} \) be the sum of the first 12 terms of the arithmetic sequence.

a. Find \( r \).

b. Show that the sum of the infinite sequence is \( 4\log_2 x \).

c. Find \( d \), giving your answer as an integer.

d. Show that \( S_{12} = 12\log_2 x - 66 \).

e. Given that \( S_{12} \) is equal to half the sum of the infinite geometric sequence, find \( x \), giving your answer in the form \( 2^p \), where \( p \in \mathbb{Q} \).
The first two terms of an infinite geometric sequence, in order, are

\[ 2\log_2 x, \log_2 x, \text{ where } x > 0. \]

a. Find \( r \).

b. Show that the sum of the infinite sequence is \( 4\log_2 x \).

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a. evidence of dividing terms (in any order) \( (M1) \)

\[ \frac{\mu_2}{\mu_1} = \frac{2\log_2 x}{\log_2 x}. \]

\[ r = \frac{1}{2} \quad A1 \quad N2 \]

[2 marks]

b. correct substitution \( (A1) \)

\[ \frac{2\log_2 x}{1 - \frac{1}{2}} \]

correct working \( A1 \)

\[ \frac{2\log_2 x}{\frac{1}{2}} \]

\[ S_\infty = 4\log_2 x \quad AG \quad N0 \]
The first three terms of an arithmetic sequence, in order, are

$$\log_2 x, \log_2 \left(\frac{x}{2}\right), \log_2 \left(\frac{x}{4}\right),$$

where $x > 0$.

Let $S_{12}$ be the sum of the first 12 terms of the arithmetic sequence.

c. Find $d$, giving your answer as an integer.

d. Show that $S_{12} = 12\log_2 x - 66$.

e. Given that $S_{12}$ is equal to half the sum of the infinite geometric sequence, find $x$, giving your answer in the form $2^p$, where $p \in \mathbb{Q}$.

d. correct substitution into the formula for the

$$eg \quad \frac{12}{2} (2\log_2 x + (12 - 1)(-1))$$

correct working $\quad A1$

g. $6(2\log_2 x - 11), \quad \frac{12}{2} (2\log_2 x - 11)$

$12\log_2 x - 66 \quad A0$ $\quad N0$

c. evidence of subtracting two terms (in any order) $\quad (M1)$

er. correct equation $\quad (A1)$

$$eg \quad 12\log_2 x - 66 = 2\log_2 x$$

correct working $\quad (A1)$

er. $10\log_2 x = 66$, $\log_2 x = 6.6$, $2^{6.6} = x^{10}$, $\log_2 \left(\frac{x^{12}}{x^2}\right) = 66$

$x = 2^{6.6}$ (accept $p = \frac{66}{10}$) $\quad A1 \quad N2$
Creating your own question

Create a question, either about

1) Amortization or 2) annuities

1 Brandt receives a loan of $10000 from a bank at an annual interest rate of 6% compounded monthly to be repaid in monthly installments within a 10-year period.
   a Determine the monthly installments in order to repay the loan on time.
   b Find how much she still owes after the fifth year.

3 Maude decides to invest in a 25-year private pension scheme, where she will have to deposit TRY1000 every month. The rate of interest is fixed at 8% per annum. Interest is compounded monthly.
   a Determine the future value of her investment at the end of the 25 years.
   b Calculate how long it will be until she breaks even with the amount that she invested?